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A VISCOELASTIC STUDY OF AN ADHESIVELY BONDED JOINT

by

Paul F. Joseph

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Applied Mechanics

Lehigh University

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Sept. 10, 1982

date

Professor in Charge

Chairman of Department

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Abstract

In this study the plane strain problem of two dissimilar orthotropic plates bonded with an isotropic, linearly viscoelastic adhesive is considered. Both the shear and the normal stresses in the adhesive are calculated for various geometries and loading conditions. Transverse shear deformations of the adherends are taken into account, and their effect on the solution is shown in the results. All three in-plane strains of the adhesive are included. Attention is given to the effect of temperature, both in the adhesive joint problem in part I and in a separate study of heat generation in a viscoelastic material under cyclic loading presented in part II. This separate study is included because heat generation and or spacially varying temperature are at present too difficult to account for in the analytical solution of the bonded joint, but whose effect can not be ignored in design.

In Part I if the temperature is taken as a known piecewise constant function of time, the differential equations have constant coefficients and the Laplace Transform technique can be directly applied. In the heat generation problem the one-dimensional coupled heat equation is solved. It is shown that the coupling term is negligible. Both experimental and theoretical results are given for various cycling frequencies.

An extension of the joint problem in Part I is a calculation from fracture mechanics of the strain energy release rate when debonding of the joint takes place. The fracture energy is found to be nearly independent of the bond length for lengths consistent with a plate theory.

Part I

The Adhesive Joint

1. Introduction

Bonding as a means of attachment and as a way for reinforcement is currently in wide use in the aerospace industry. Its main mechanical advantage over riveting is that the load is carried over a larger area, thus reducing the stress concentration. Another advantage is that no holes are required which favors the use of high strength, low weight fiber reinforced composites. Indeed the development of these materials is achieved through a bonding process.

However, bonding of joints has its own problems. Unfortunately the load is not carried over the entire bond area, but instead is confined to a small region along the bond edge. This highly stressed region, though not as high as the stresses at a rivet, can lead to one of several modes of failure. First consider the failure of the adherends (for geometry of the joint see figures 1a,b). At the edge of the bond region there are very high stresses in both the adherends and the adhesive. In linear elasticity these stresses are actually singular (see [1],[2]). However, because of the geometrical complexities involved in an adhesive joint - the combining of three distinct materials - several simplifications of the three-dimensional elasticity problem are made. The adhesive is modeled as a tension, shear spring by averaging all stresses and strains through the thickness and the adherends are modeled as plates. Therefore these singular stresses are not observed, and it can be shown that all stresses are bounded. It is of interest to note that even if the thickness variation of stresses in the adhesive is ignored, the normal stress in the x-direction in the adherend will have a logarithmic singularity at the bond edge. This results from the discontinuous shear traction acting on the surface of the adherend. The normal stress in the adhesive does not cause any singular stresses in the adherends (see [3]). Due to these high stresses, the adherends could fail either by yielding of the material or by some form of material separation such as cracking in the case of isotropic adherends, or delamination in the case of laminated adherends. Cracking would probably be attributed to the shear stress; delamination or transverse pulling apart of the fiber layers is most likely the result of the normal stress. Yielding could be attributed to both stresses.

In order to analyze the failure of the adherends, one should treat them as elastic continua. In this and most other studies, the adherends are modeled as plates, and therefore the high singular stress region in the adherend at the edge of the bond is not observed. There are several papers that treat either one or both of the adherends as elastic continua [4-6]. However, in [6] it is found that there are severe convergence problems when the adherends are relatively thin and this is precisely the geometry when adherend failure becomes dominant as pointed out in [7]. It is possible to analyse failure of the adherends if the bending stresses due to eccentricity of the load path are taken into account as was first investigated in [8] (see figures 2a,b). This involves determination of the loading condition in figure 2b in terms of the loading and geometry of figure 2a. Equilibrium must actually be considered in the loaded position and therefore this is a nonlinear procedure. In this study the loads of figure 2b are assumed known.

If the adherends are thick enough so that adherend failure is unlikely, cracking or peeling of the adhesive may result due to high shear and normal stresses at the bond edge. This is a mixed mode fracture mechanics problem where the shear stress can be more important than the normal stress. In this study the strain energy release rate is calculated, which may be used as the measure of the magnitude of the external loads and the severity of joint geometry in fatigue and fracture analysis.

Most of the effort in the literature has been devoted to the calculation of the adhesive stresses. It is in the constitutive modeling of the adhesive that the various investigations differ. They vary from elastic to a nonlinear viscoelastic behavior [2]. It is true that the epoxy which is subjected to such high stresses at the bond edge will not behave in a linear way. An elastic-plastic modeling of the adhesive is perhaps the simplest way to incorporate this nonlinearity of material behavior. However, the analytical solution of such a formulation is very complicated (see for example [1]). A viscoplastic solution, which incorporates all other mentioned theories, is better still but an accurate analysis requires a purely numerical technique such as finite elements.

The analytical solution presented in this study uses a linear viscoelastic modeling of the adhesive. The hereditary integral formulation is used and therefore the model is an accurate one. It requires the relaxation modulus in shear which can be any function, and, for practical applications, can be obtained from a fit to the experimental data. The second material "constant" needed to define an isotropic material is the bulk modulus which is assumed to be time independent. This means that under a hydrostatic state of stress the material behaves elastically. It is an assumption which is quite commonly made. A check of this assumption was performed using experimental data for an epoxy resin, Hercules 3501-5A. This data was obtained from [9]. They fit curves to data for both the relaxation modulus in shear ($G(t)$) and in tension ($E(t)$).

A calculated value of the bulk modulus was not time independent and varied as much as E and G. However, the time independence of K is believed to be valid for most materials.

As far as the plate modeling is concerned, it is generally accepted that transverse shear deformations should be taken into account because of the high stresses involved. In this study this addition involved very little extra algebra because the problem was solved under the plane strain assumption. Also the order of the differential equations for the stresses was not increased. The inclusion of any extra degree of freedom for the plate beyond what is provided by the Classical theory will probably have some affect on the stresses. A more advanced plate theory was used in [10] where the strain in the normal direction to the plate was non-zero. At the bond edge one can imagine a pinching effect to exist making this quantity nonnegligible. Apparently this addition changes the order of the differential equation and there is a requirement for an "extra" boundary condition. The researchers of [10] forced the shear stress to be zero at the bond edge (i.e., $\tau=0$ at $x=\pm\ell$). Since the stresses in the adhesive layer are averaged through the thickness, one can not specify an elasticity boundary condition and ignore the corners of the adhesive where the stresses are singular. Perhaps another boundary condition could be employed (see [1]).

The problem considered in this investigation is a further generalization of work done by F. Delale and F. Erdogan [1,11,12]. It was in [11] when they presented the viscoelastic solution for identical adherends. In [1] they were joined by M.N. Ayduroglu to publish a paper on the general elastic closed form solution with a finite element check of their results. It was shown in this report that within geometrical restrictions the plate theory gives good results for the normal and shear stress in the adhesive. The restrictions are roughly that the ratio of adherend thickness to adhesive thickness should be approximately an order of magnitude and the ratio of bond length to adherend thickness should also be an order of magnitude. Then in [12] further research by Delale and Erdogan included the influence of temperature on the adhesive and how it affects the stresses. Here the adherends were identical and therefore no thermal stresses were present. In this study the problem with dissimilar orthotropic adherends is considered. This change, besides including thermal stresses, makes the solution useful.

2. Formulation of the Problem

The problem considered is either the single lap joint (figure 1a) or the cover plate (figure 1b). A plate theory is used taking into account transverse shear deformations. Also the problem will be solved under the plane strain or cylindrical bending assumption which requires that the geometry and loading are constant in the z-direction. The only independent spacial variable is x. The viscoelastic nature of the adherend also make time "t" an independent variable.

Equilibrium of the element shown in figure 3a gives the following relationships:

$$\frac{\partial N_{1x}}{\partial x} = \tau \qquad \frac{\partial N_{2x}}{\partial x} = -\tau \tag{1a,b}$$

$$\frac{\partial Q_{1x}}{\partial x} = \sigma \qquad \frac{\partial Q_{2x}}{\partial x} = -\sigma \tag{2a,b}$$

$$\frac{\partial M_{1x}}{\partial x} = Q_{1x} - \frac{h_1+h_0}{2}\tau \qquad \frac{\partial M_{2x}}{\partial x} = Q_{2x} - \frac{h_2+h_0}{2}\tau\ . \tag{3a,b}$$

N_{ix}, Q_{ix}, and M_{ix} (i=1,2) are respectively the resultant normal force, resultant shear force, and resultant bending moment in the adherends. The adhesive stresses are τ, the shear stress, and σ, the transverse normal stress also shown in figure 3a.

Taking $(T-T_0)H(t-t_2)$ as the temperature function where $T-T_0$ is a constant and $H(t)$ is the unit step function, the stress resultant-displacement relations for the adherends are:

$$\frac{\partial u_i}{\partial x} = C_i N_{ix} + (\alpha_{ix} + \alpha_{iz}\nu_{ixz})(T-T_0)H(t-t_2) \quad i=1,2\ , \tag{4a,b}$$

$$\frac{\partial \beta_i}{\partial x} = D_i M_{ix} \qquad i = 1,2 \;, \tag{5a,b}$$

$$\frac{\partial v_i}{\partial x} + \beta_i = Q_{ix}/B_i \qquad i = 1,2 \;, \tag{6a,b}$$

where

$$C_i = \frac{1-\nu_{ix}\nu_{iz}}{E_{ix}h_i} \qquad D_i = \frac{12(1-\nu_{ix}\nu_{iz})}{h_i{}^3E_{ix}} \qquad B_i = \frac{5}{6}\, h_i G_{ixy} \tag{7}$$

and u and v are the x and y components of displacement of the mid-plane of the adherends and β is the rotation of the normal. Note that the term Q_{ix}/B_i includes the effect of transverse shear.

Since the adhesive is thin compared to the adherends, the average values of the strains are used - i.e. the y variation is neglected. See figure 1c for these kinematical considerations.

$$\begin{aligned} \varepsilon_y &= (v_1-v_2)/h_o \\ \varepsilon_x &= \left(\frac{\partial u_1}{\partial x} - \frac{h_1}{2}\frac{\partial \beta_{1x}}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{h_2}{2}\frac{\partial \beta_{2x}}{\partial x}\right)/2 \\ \gamma_{xy} &= (u_1 - \frac{h_1}{2}\beta_{1x} - u_2 - \frac{h_2}{2}\beta_{2x})/h_o \;. \end{aligned} \tag{8a,b,c}$$

The hereditary integral approach will be used to model the adhesive. For a linear, isotropic, viscoelastic adhesive we can write:

$$s_{ij} = 2\int_{-\infty}^{t} G(T,t-\xi)\,\frac{\partial e_{ij}}{\partial \xi}\,d\xi \qquad (i,j=x,y,z) \;, \tag{9}$$

$$e = \frac{s}{3K(T)} + \alpha_3(T)(T-T_0)H(t-t_2) \tag{10}$$

where

$$e = (\varepsilon_x+\varepsilon_y+\varepsilon_z)/3 \ , \ e_{ij} = \varepsilon_{ij} - e\delta_{ij} \quad (i,j=x,y,z) \tag{11}$$

and

$$s = (\sigma_x+\sigma_y+\sigma_z)/3 \ , \ s_{ij} = \sigma_{ij} - s\delta_{ij} \quad (i,j=x,y,z) \ . \tag{12}$$

In the adhesive the only non-zero stresses are σ_{xx}, $\tau_{xy} = \tau$, $\sigma_{yy} = \sigma$, and σ_z and the only non-zero strains are ε_x, ε_y, and γ_{xy}. Substitution of (11) and (12) into (9) and (10) taking into account the preceeding, we get:

$$2\sigma_x-\sigma-\sigma_z = 2\int_{-\infty}^{t} G(T,t-\xi)(2\frac{\partial\varepsilon_x}{\partial\xi} - \frac{\partial\varepsilon_y}{\partial\xi})d\xi \tag{13}$$

$$2\sigma-\sigma_x-\sigma_z = 2\int_{-\infty}^{t} G(T,t-\xi)(2\frac{\partial\varepsilon_y}{\partial\xi} - \frac{\partial\varepsilon_x}{\partial\xi})d\xi \tag{14}$$

$$2\sigma_z-\sigma_x-\sigma = -2\int_{-\infty}^{t} G(T,t-\xi)(\frac{\partial\varepsilon_x}{\partial\xi} + \frac{\partial\varepsilon_y}{\partial\xi})d\xi \tag{15}$$

$$\tau = \int_{-\infty}^{t} G(T,t-\xi)\frac{\partial\gamma_{xy}}{\partial\xi}d\xi \tag{16}$$

$$\sigma_x + \sigma + \sigma_z = 3K(T)[\varepsilon_x+\varepsilon_y - 3\alpha_3(T)(T-T_0)H(t-t_2)] \ . \tag{17}$$

Since $\Sigma s_{ii} = 0$ and $\Sigma e_{ii} = 0$, equations (13-15) are linearly dependent; (14), for example, can be obtained by adding (13) and (15), so it is ignored. Eliminating σ_x and σ_z from (13), (15), and (17), it follows that

$$\sigma = K(T)(\varepsilon_x+\varepsilon_y) - 3K(T)\alpha_3(T)(T-T_0)H(t-t_2)$$
$$- \frac{2}{3}\int_{-\infty}^{t} G(T,t-\xi)\left(\frac{\partial\varepsilon_x}{\partial\xi} - 2\frac{\partial\varepsilon_y}{\partial\xi}\right)d\xi \quad . \tag{18}$$

Equations (1-6), (16) and (18) now make it possible to solve for the unknowns σ, τ, N_{ix}, Q_{ix}, M_{ix}, u_i, v_i, and β_i.

3. The General Solution

We are interested mainly in σ and τ so the other variables are eliminated through algebra as follows.

Differentiate equations (5a,b) three times with respect to x and make use of relations (3) and (2) to obtain

$$\frac{\partial^3\beta_{1x}}{\partial x^3} = D_1\left(\sigma - \frac{h_1+h_o}{2}\frac{\partial\tau}{\partial x}\right) \quad , \tag{19a}$$

$$\frac{\partial^3\beta_{2x}}{\partial x^3} = -D_2\left(\sigma + \frac{h_2+h_o}{2}\frac{\partial\tau}{\partial x}\right) \quad . \tag{19b}$$

Next take equation (16) and substitute for γ_{xy}.

$$\tau = \frac{1}{h_o}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial\xi}\left(u_1 - \frac{h_1}{2}\beta_{1x} - u_2 - \frac{h_2}{2}\beta_{2x}\right)d\xi \quad . \tag{20}$$

Differentiate once with respect to x

$$\frac{\partial\tau}{\partial x} = \frac{1}{h_o}\int_{-\infty}^{t} G(T,t-\xi)\,\frac{\partial}{\partial\xi}\left(\frac{\partial u_1}{\partial x} - \frac{h_1}{2}\frac{\partial\beta_{1x}}{\partial x} - \frac{\partial u_2}{\partial x} - \frac{h_2}{2}\frac{\partial\beta_{2x}}{\partial x}\right)d\xi\,, \tag{21}$$

again

$$\frac{\partial^2\tau}{\partial x^2} = \frac{1}{h_o}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial\xi}\left(\frac{\partial^2 u_1}{\partial x^2} - \frac{h_1}{2}\frac{\partial^2\beta_{1x}}{\partial x^2} - \frac{\partial^2 u_2}{\partial x^2} - \frac{h_2}{2}\frac{\partial^2\beta_{2x}}{\partial x^2}\right)d\xi\,. \tag{22}$$

Next use relations(1) and (4) to substitute for $\frac{\partial^2 u_i}{\partial x^2}$.

$$\frac{\partial^2\tau}{\partial x^2} = \frac{1}{h_o}\int_{-\infty}^{t} G(T,t-\xi)\,\frac{\partial}{\partial\xi}\left(C_1\tau - \frac{h_1}{2}\frac{\partial^2\beta_{1x}}{\partial x^2} + C_2\tau - \frac{h_2}{2}\frac{\partial^2\beta_{2x}}{\partial x^2}\right)d\xi\,. \tag{23}$$

Differentiate once more and use (19a,b)

$$\frac{\partial^3\tau}{\partial x^3} = \frac{1}{h_o}\int_{-\infty}^{t} G(T,t-\xi)\left[A_1\,\frac{\partial^2\tau}{\partial x\partial\xi} + A_2\,\frac{\partial\sigma}{\partial\xi}\right]d\xi\,, \tag{24}$$

where

$$A_1 = C_1 + C_2 + D_1\,\frac{h_1(h_1+h_o)}{4} + D_2\,\frac{h_2(h_2+h_o)}{4} \tag{25}$$

$$A_2 = -\frac{D_1 h_1}{2} + \frac{D_2 h_2}{2}\,. \tag{26}$$

Next use equation (18) and make substitutions for the strains (8a,b,c).

$$\sigma = K(T)[(v_1-v_2)/h_o + \frac{1}{2}(\frac{\partial u_1}{\partial x} - \frac{h_1}{2}\frac{\partial \beta_{1x}}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{h_2}{2}\frac{\partial \beta_{2x}}{\partial x})]$$

$$- 3K(T)\alpha_3(T)(T-T_o)H(t-t_2)$$

$$- \frac{2}{3}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial \xi}[\frac{1}{2}(\frac{\partial u_1}{\partial x} - \frac{h_1}{2}\frac{\partial \beta_{1x}}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{h_2}{2}\frac{\partial \beta_{2x}}{\partial x})$$

$$- \frac{2}{h_o}(v_1-v_2)]d\xi \; . \tag{27}$$

Differentiate once with respect to x and substitute using (6) and (4) together with (1)

$$\frac{\partial \sigma}{\partial x} = K(T)[\frac{1}{h_o}(\frac{Q_{1x}}{B_1} - \beta_{1x} - \frac{Q_{2x}}{B_2} + \beta_{2x}) + \frac{1}{2}(C_1\tau - C_2\tau$$

$$- \frac{h_1}{2}\frac{\partial^2 \beta_{1x}}{\partial x^2} + \frac{h_2}{2}\frac{\partial^2 \beta_{2x}}{\partial x^2})$$

$$- \frac{2}{3}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial \xi}[\frac{1}{2}(C_1\tau - C_2\tau - \frac{h_1}{2}\frac{\partial^2 \beta_{1x}}{\partial x^2} + \frac{h_2}{2}\frac{\partial^2 \beta_{2x}}{\partial x^2})$$

$$- \frac{2}{h_o}(\frac{Q_{1x}}{B_1} - \beta_{1x} - \frac{Q_{2x}}{B_2} + \beta_{2x})]d\xi \; . \tag{28}$$

Differentiate again using (2), (10)

$$\frac{\partial^2\sigma}{\partial x^2} = K(T)[\frac{1}{h_o}(\frac{1}{B_1}\sigma - \frac{\partial\beta_{1x}}{\partial x} + \frac{1}{B_2}\sigma + \frac{\partial\beta_{2x}}{\partial x})$$

$$+ \frac{1}{2}[(C_1-C_2)\frac{\partial\tau}{\partial x} - D_1\frac{h_1}{2}(\sigma - \frac{h_1+h_o}{2}\frac{\partial\tau}{\partial x}) - D_2\frac{h_2}{2}(\sigma + \frac{h_2+h_o}{2}\frac{\partial\tau}{\partial x})]]$$

$$- \frac{2}{3}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial\xi}[\frac{1}{2}(C_1\frac{\partial\tau}{\partial x} - C_2\frac{\partial\tau}{\partial x} - D_1\frac{h_1}{2}(\sigma - \frac{h_1+h_o}{2}\frac{\partial\tau}{\partial x})$$

$$-D_2\frac{h_2}{2}(\sigma + \frac{h_2+h_o}{2}\frac{\partial\tau}{\partial x})) - \frac{2}{h_o}(\frac{\sigma}{B_1} - \frac{\partial\beta_{1x}}{\partial x} + \frac{\sigma}{B_2} + \frac{\partial\beta_{2x}}{\partial x})]d\xi \;. \tag{29}$$

Differentiate once more

$$\frac{\partial^3\sigma}{\partial x^3} = K(T)[\frac{1}{h_o}(\frac{1}{B_1}\frac{\partial\sigma}{\partial x} - \frac{\partial^2\beta_{1x}}{\partial x^2} + \frac{1}{B_2}\frac{\partial\sigma}{\partial x} + \frac{\partial^2\beta_{2x}}{\partial x^2})$$

$$+ \frac{1}{2}[(C_1-C_2)\frac{\partial^2\tau}{\partial x^2} - D_1\frac{h_1}{2}(\frac{\partial\sigma}{\partial x} - \frac{h_1+h_o}{2}\frac{\partial^2\tau}{\partial x^2}) - D_2\frac{h_2}{2}(\frac{\partial\sigma}{\partial x} + \frac{h_2+h_o}{2}\frac{\partial^2\tau}{\partial x^2})]]$$

$$- \frac{2}{3}\int_{-\infty}^{t} G(T,t-\xi)\frac{\partial}{\partial\xi}[\frac{1}{2}(C_1\frac{\partial^2\tau}{\partial x^2} - C_2\frac{\partial^2\tau}{\partial x^2} - D_1\frac{h_1}{2}(\frac{\partial\sigma}{\partial x} - \frac{h_1+h_o}{2}\frac{\partial^2\tau}{\partial x^2})$$

$$-D_2\frac{h_2}{2}(\frac{\partial\sigma}{\partial x} + \frac{h_2+h_o}{2}\frac{\partial^2\tau}{\partial x^2})) - \frac{2}{h_o}(\frac{1}{B_1}\frac{\partial\sigma}{\partial x} - \frac{\partial^2\beta_{1x}}{\partial x^2} + \frac{1}{B_2}\frac{\partial\sigma}{\partial x} + \frac{\partial^2\beta_{2x}}{\partial x^2})]d\xi. \tag{30}$$

Differentiate again making use of (19)

$$\frac{\partial^4\sigma}{\partial x^4} = K(T)[f_1 \frac{\partial^2\sigma}{\partial x^2} + f_2\sigma + f_3 \frac{\partial^3\tau}{\partial x^3} + f_4 \frac{\partial\tau}{\partial x}]$$

$$- \frac{2}{3} \int_{-\infty}^{t} G(T,t-\xi) \frac{\partial}{\partial\xi}[f_5 \frac{\partial^2\sigma}{\partial x^2} -2f_2\sigma + f_3 \frac{\partial^3\tau}{\partial x^3} -2f_4 \frac{\partial\tau}{\partial x}]d\xi, \tag{31}$$

where

$$f_1 = \frac{1}{h_oB_1} + \frac{1}{h_oB_2} - \frac{D_1h_1}{4} - \frac{D_2h_2}{4} \tag{32}$$

$$f_2 = -(D_1+D_2)/h_o \tag{33}$$

$$f_3 = \frac{C_1}{2} - \frac{C_2}{2} + \frac{D_1h_1(h_1+h_o)}{8} - \frac{D_2h_2(h_2+h_o)}{8} \tag{34}$$

$$f_4 = \frac{D_1(h_1+h_o)}{2h_o} - \frac{D_2(h_2+h_o)}{2h_o} \tag{35}$$

$$f_5 = -[\frac{2}{h_oB_1} + \frac{2}{h_oB_2} + \frac{D_1h_1}{4} + \frac{D_2h_2}{4}] . \tag{36}$$

Now assume that the temperature level is suddenly fixed at 0^- with joint stress free. Now take Laplace Transform of equations (24), and (31)

$$\frac{\partial^3\bar{\tau}}{\partial x^3} = \frac{1}{h_o} \bar{G}(A_1 s \frac{\partial\bar{\tau}}{\partial x} + A_2 s\bar{\sigma}) \tag{37}$$

$$\frac{\partial^4\bar{\sigma}}{\partial x^4} = K(T)[f_1 \frac{\partial^2\bar{\sigma}}{\partial x^2} + f_2\bar{\sigma} + f_3 \frac{\partial^3\bar{\tau}}{\partial x^3} + f_4 \frac{\partial\bar{\tau}}{\partial x}]$$

$$- \frac{2}{3} \bar{G}[f_5 s \frac{\partial^2\sigma}{\partial x^2} - 2f_2 s\bar{\sigma} + f_3 s \frac{\partial^3\bar{\tau}}{\partial x^3} - 2f_4 s \frac{\partial\bar{\tau}}{\partial x}] \ . \tag{38}$$

Now solving (37) for $\bar{\sigma}$

$$\bar{\sigma} = a_7 \frac{\partial^3\bar{\tau}}{\partial x^3} + a_8 \frac{\partial\bar{\tau}}{\partial x} \ , \tag{39}$$

where

$$a_7 = \frac{h_o}{\bar{G}s} \frac{2}{(-D_1 h_1 + D_2 h_2)} \qquad a_8 = - \frac{A_1}{A_2} \ . \tag{40}$$

Rearranging (38)

$$\frac{\partial^4\bar{\sigma}}{\partial x^4} = a_3 \frac{\partial^2\bar{\sigma}}{\partial x^2} + a_4\bar{\sigma} + a_5 \frac{\partial^3\bar{\tau}}{\partial x^3} + a_6 \frac{\partial\bar{\tau}}{\partial x} \ , \tag{41}$$

substituting (39) into (41) we obtain

$$\frac{\partial^7\bar{\tau}}{\partial x^7} + C_1 \frac{\partial^5\bar{\tau}}{\partial x^5} + C_2 \frac{\partial^3\bar{\tau}}{\partial x^3} + C_3 \frac{\partial\bar{\tau}}{\partial x} = 0 \tag{42}$$

where

$$C_1 = \frac{a_8 - a_3 a_7}{a_7} \qquad C_2 = \frac{-a_3 a_8 - a_4 a_7 - a_5}{a_7}$$

$$C_3 = \frac{-a_4 a_8 - a_6}{a_7} \tag{43}$$

and

$$a_1 = \frac{1}{h_o} \bar{G}(T,s)[C_1+C_2+D_1 \frac{h_1(h_1+h_o)}{4} + D_2 \frac{h_2(h_o+h_2)}{4}]$$

$$a_2 = \frac{1}{h_o} \bar{G}(T,s)[\frac{-D_1h_1}{2} + \frac{D_2h_2}{2}]$$

$$a_3 = K(T)[\frac{1}{h_oB_1} + \frac{1}{h_oB_2} - \frac{D_1h_1}{4} - \frac{D_2h_2}{4}]$$

$$+ \frac{2}{3} \bar{G}(T,s)s[\frac{D_1h_1}{4} + \frac{D_2h_2}{4} + \frac{2}{h_oB_1} + \frac{2}{h_oB_2}]$$

$$a_4 = [K(T) + \frac{4}{3} s \bar{G}(T,s)][- \frac{D_1}{h_o} - \frac{D_2}{h_o}]$$

$$a_5 = [K(T)- \frac{2}{3} s \bar{G}(T,s)][\frac{C_1-C_2}{2} + \frac{D_1h_1(h_o+h_1)}{8} - \frac{D_2h_2(h_o+h_2)}{8}]$$

$$a_6 = [K(T) + \frac{4}{3} s \bar{G}(T,s)][\frac{D_1(h_o+h_1)}{2h_o} - \frac{D_2(h_o+h_2)}{2h_o}]$$

$$a_7 = \frac{1}{sa_2}$$

$$a_8 = - \frac{a_1}{a_2} \quad . \tag{44}$$

Since the coefficients in equation (42) are not x dependent, we look for a solution of the form e^{mx}.

The characteristic equation becomes

$$m^7 + c_1m^5 + c_2m^3 + c_3m = 0 \quad . \tag{45}$$

Say this equation has the roots

$$0,\ \pm\gamma_1,\ \pm\gamma_2,\ \pm\gamma_3 \tag{46}$$

where γ_1, γ_2 and γ_3 are the roots of

$$\gamma^3 + c_1\gamma^2 + c_2\gamma + c_3 = 0\ . \tag{47}$$

The solution is then

$$\bar{\tau}(x,s) = A_0+A_1\sinh\gamma_1 x+A_2\cosh\gamma_1 x+A_3\sinh\gamma_2 x + A_4\cosh\gamma_2 x$$

$$+ A_5\sinh\gamma_3 x + A_6\cosh\gamma_3 x \tag{48}$$

which may be written as

$$\bar{\tau}(x,s) = A_0 + \sum_{i=1}^{3} (A_{2i-1}\ \sinh\gamma_i x + A_{2i}\ \cosh\gamma_i x)\ . \tag{49}$$

From (28) we find

$$\bar{\sigma}(x,s) = \sum_{i=1}^{3} (A_{2i-1}(a_7\gamma_i{}^3+a_8\gamma_i)\cosh\gamma_i x + A_{2i}(a_7\gamma_i{}^3+a_8\gamma_i)\sinh\gamma_i x). \tag{50}$$

The constants A_i $(i=0,\ldots,6)$ are determined from the boundary conditions. The seven relations to be used to obtain these constants are the second and third derivatives of equation (18) (equations 29 and 30), the first and second derivatives of equation (16) (equations 21 and 22), and the following three relations which refer to figure 3b.

$$\int_{-\ell}^{\ell} \tau(x,t)dx = N_2(-\ell)H(t-t_1)-N_0\ H(t-t_1) \tag{51}$$

$$\int_{-\ell}^{\ell} \sigma(x,t)dx = Q_2(-\ell)H(t-t_1)-Q_oH(t-t_1) \tag{52}$$

$$\int_{-\ell}^{\ell} x\sigma(x,t)dx = [M_o-M_2(-\ell)+N_2(-\ell)\ \frac{h_o+h_2}{2} - \ell Q_o - \ell Q_2(-\ell)$$

$$- N_o\ \frac{h_o+h_2}{2}\ ]H(t-t_1)\ . \tag{53}$$

Now the Laplace Transform of these seven expressions must be taken. They become

$$\frac{\partial^2\bar{\sigma}}{\partial x^2} = \bar{\sigma}[K(T)(\frac{1}{h_oB_1} + \frac{1}{h_oB_2} - \frac{D_1h_1}{4} - \frac{D_2h_2}{4}$$

$$- \frac{2}{3}\ s\ \bar{G}(T,s)(- \frac{D_1h_1}{4} - \frac{D_2h_2}{4} - \frac{2}{h_oB_1} - \frac{2}{h_oB_2}$$

$$+ \frac{\partial\bar{\tau}}{\partial x}\ [(K(T)- \frac{2}{3}\ s\ \bar{G}(T,s))(\frac{D_1h_1}{8}(h_1+h_o)- \frac{D_2h_2}{8}(h_2+h_o))+ \frac{1}{2}(c_1+c_2))]$$

$$+ [K(T)(- \frac{D_1\bar{M}_{1x}}{h_o} + \frac{D_2\bar{M}_{2x}}{h_o})+ \frac{4}{3}\ \bar{G}(T,s)s(- \frac{D_1\bar{M}_{1x}}{h_o} + \frac{D_2\bar{M}_{2x}}{h_o})], \tag{54}$$

and

$$\frac{\partial^3\bar{\sigma}}{\partial x^3} = \frac{\partial\bar{\sigma}}{\partial x} [K(T)(\frac{1}{h_oB_1} + \frac{1}{h_oB_2} - \frac{D_1h_1}{4} - \frac{D_2h_2}{4}) - \frac{2}{3} s \bar{G}(T,s)$$

$$x (- \frac{D_1h_1}{4} - \frac{D_2h_2}{4} - \frac{2}{h_oB_1} - \frac{2}{h_oB_2})]$$

$$+ \frac{\partial^2\bar{\tau}}{\partial x^2} [(K(T) - \frac{2}{3} s\bar{G}(T,s))(\frac{D_1h_1}{8} (h_1+h_o) - \frac{D_2h_2}{8} (h_2+h_o)) + \frac{1}{2}(c_1-c_2))]$$

$$+ [K(T)(- \frac{D_1}{h_o} \quad (\bar{Q}_{1x} - \frac{h_1+h_o}{2} \bar{\tau}) + \frac{D_2}{h_o} (\bar{Q}_{2x} - \frac{h_2+h_o}{2} \bar{\tau}))$$

$$+ \frac{4}{3} \bar{G}(T,s)s(- \frac{D_1}{h_o}(\bar{Q}_{1x} - \frac{h_1+h_o}{2} \bar{\tau}) + \frac{D_2}{h_o} (\bar{Q}_{2x} - \frac{h_2+h_o}{2} \bar{\tau}))]$$

(55)

$$\frac{\partial\bar{\tau}}{\partial x} = \frac{\bar{G}s}{h_o} [C_1\bar{N}_{1x} + (\alpha_{1x}+\alpha_{1z}\nu_{1xz})(T-T_o) \frac{e^{-st_2}}{2} - \frac{h_1}{2} D_1\bar{M}_{1x}$$

$$- C_2\bar{N}_{2x} - (\alpha_{2x}+\alpha_{2z}\nu_{2xz})(T-T_o) \frac{e^{-st_2}}{s} - \frac{h_2}{2} D_2\bar{M}_{2x}] \qquad (56)$$

$$\frac{\partial^2\tau}{\partial x^2} = \frac{\bar{G}s}{h_o} [C_1\bar{\tau} - \frac{h_1D_1}{2} (\bar{Q}_{1x} - \frac{h_1+h_o}{2} \bar{\tau}) + C_2\bar{\tau}$$

$$- \frac{h_2D_2}{2} (\bar{Q}_{2x} - \frac{h_2+h_o}{2} \bar{\tau})] \qquad (57)$$

$$\int_{-\ell}^{\ell} \bar{\sigma}\, dx = Q_2(-\ell) \frac{e^{-st_1}}{s} - Q_o \frac{e^{-st_1}}{s} \qquad (58)$$

$$\int_{-\ell}^{\ell} \bar{\tau}\, dx = N_2(-\ell)\,\frac{e^{-st_1}}{s} - N_o\,\frac{e^{-st_1}}{s} \tag{59}$$

$$\int_{-\ell}^{\ell} x\bar{\sigma}\, dx = [M_o - M_2(-\ell) + N_2(-\ell)\,\frac{h_o+h_2}{2} - \ell\, Q_o - \ell\, Q_2(-\ell) - N_o\,\frac{h_o+h_2}{2}]\,\frac{e^{-st_1}}{s}. \tag{60}$$

Now substituting (49) and (50) into (54-60), and then evaluating (54-57) at $x=\ell$ and integrating (58-60) we get the following 7 equations which are sufficient to determine the 7 unknown constants A_i $(i=0,1,\ldots,6)$.

$$\sum_{i=1}^{3}[A_{2i-1}(a_7\gamma_i^5+\rho_1\gamma_i^3+\rho_2\gamma_i)\cosh\gamma_i\ell$$

$$+ A_{2i}(a_7\gamma_i^5+\rho_1\gamma_i^3+\rho_2\gamma_i)\sinh\gamma_i\ell]$$

$$= (K(T) + \frac{4}{3}\,\bar{G}s)[-\frac{D_1M_1(\ell)}{h_os} + \frac{D_2M_o}{h_os}]e^{-st_1} \tag{61}$$

$$A_o\rho_3 + \sum_{i=1}^{3}[A_{2i-1}(a_7\gamma_i^6+\rho_1\gamma_i^4+\rho_2\gamma_i^2+\rho_3)\sinh\gamma_i\ell$$

$$+ A_{2i}[a_7\gamma_i^6+\rho_1\gamma_i^4+\rho_2\gamma_i^2+\rho_3)\cosh\gamma_i\ell]$$

$$= [K(T) + \frac{4}{3}\,\bar{G}s][-\frac{D_1Q_1(\ell)}{h_os} + \frac{D_2Q_o}{h_os}]e^{-st_1} \tag{62}$$

$$\sum_{i=1}^{3}[A_{2i-1}\gamma_i\cosh\gamma_i\ell + A_{2i}\gamma_i\sinh\gamma_i\ell]$$

$$= \frac{1}{h_o}\bar{G}s[C_1\frac{N_1(\ell)}{s}e^{-st_1}+(\alpha_{1x}+\alpha_{1z}\nu_{1xz})(T-T_o)\frac{e^{-st_2}}{s} - \frac{h_1}{2}D_1\frac{M_1(\ell)}{s}e^{-st_1}$$

$$- C_2\frac{N_o}{s}e^{-st_1}-(\alpha_{2x}+\alpha_{2z}\nu_{2xz})(T-T_o)\frac{e^{-st_2}}{s} - \frac{h_2}{2}D_2\frac{M_o}{s}e^{-st_1}$$

$$(63)$$

$$A_o\rho_4 + \sum_{i=1}^{3}[A_{2i-1}(\gamma_i^2+\rho_4)\sinh\gamma_i\ell+A_{2i}(\gamma_i^2+\rho_4)\cosh\gamma_i\ell]$$

$$= \frac{1}{h_o}\bar{G}s[-\frac{h_1D_1}{2}\frac{Q_o}{s}e^{-st_1} - \frac{h_2D_2}{2}\frac{Q_2(\ell)}{s}e^{-st_1}] \qquad (64)$$

$$2\ell A_o + 2\sum_{i=1}^{3}A_{2i}\frac{\sinh\gamma_i\ell}{\gamma_i} = \frac{N_2(-\ell)}{s}e^{-st_1} - \frac{N_o}{s}e^{-st_1} \qquad (65)$$

$$2\sum_{i=1}^{N}A_{2i-1}(a_7\gamma_i^2+a_8)\sinh\gamma_i\ell = Q_2(-\ell)\frac{e^{-st_1}}{s} - Q_o\frac{e^{-st_1}}{s} \qquad (66)$$

$$\sum_{i=1}^{3}A_{2i}[(2\ell a_7\gamma_i^2+2\ell a_8)\cosh\gamma_i\ell - (2a_7\gamma_i+2a_8\frac{1}{\gamma_i})\sinh\gamma_i\ell]$$

$$= [M_o-M_2(-\ell)+N_2(-\ell)\frac{h_o+h_2}{2} - \ell Q_o-\ell Q_2(-\ell)-N_o\frac{h_o+h_2}{2}]\frac{e^{-st_1}}{s}$$

$$(67)$$

where

$$\rho_1 = a_8 - a_7[K(T)(\frac{1}{h_oB_1} + \frac{1}{h_oB_2} - \frac{D_1h_1}{4} - \frac{D_2h_2}{4}$$

$$+ \frac{2}{3} s\bar{G}(\frac{D_1h_1}{4} + \frac{D_2h_2}{4} + \frac{2}{h_oB_1} + \frac{2}{h_oB_2})]$$

$$\rho_2 = (\frac{\rho_1 - a_8}{a_7})a_8 - (K(T) - \frac{2}{3}\bar{G}s)[\frac{D_1h_1}{8}(h_1+h_o) - \frac{D_2h_2}{8}(h_2+h_o)$$

$$+ \frac{1}{2}(C_1 - C_2)]$$

$$\rho_3 = [K(T) + \frac{4}{3}\bar{G}s][-\frac{D_1}{2h_o}(h_1+h_o) + \frac{D_2}{2h_o}(h_2+h_o)]$$

$$\rho_4 = -\frac{1}{h_o}\bar{G}s[C_1 + C_2 + \frac{h_1D_1}{4}(h_1+h_o) + \frac{h_2D_2}{4}(h_2+h_o)] \quad . \qquad (68)$$

Now that we are able to obtain the constants $A_o,\ldots,A_6$ numerically, we know $\bar{\tau}(x,s)$ and $\bar{\sigma}(x,s)$ as given by 49 and 50. We must perform the inversion to get the desired functions $\tau(x,t)$ and $\sigma(x,t)$.

$$\tau(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\tau}(x,s)e^{st}ds \qquad (69)$$

$$\sigma(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\sigma}(x,s)e^{st}ds \ . \qquad (70)$$

To perform this integration, which must be done numerically, one first must investigate the singular nature of the integrand. It is found that there is a simple pole at zero and all other singularities lie in the left half plane. Therefore, any positive value of C will be adequate.

We make the variable change s=c+iy and write the Laplace integral as a Fourier integral. Doing this we get

$$\tau(x,t)=\frac{1}{2\pi}\int_0^\infty \bar{\tau}(x,c+iy)e^{(c+iy)t}dy+\frac{1}{2\pi}\int_0^\infty \bar{\tau}(x,c-iy)e^{(c-iy)t}dy. \tag{71}$$

To evaluate this infinite integral we separate it into two integrals in order to use an asymptotic analysis.

$$\tau(x,t)=\frac{1}{2\pi}\int_0^A [\bar{\tau}(x,c+iy)e^{(c+iy)t}+\bar{\tau}(x,c-iy)e^{(c-iy)t}]dy$$

$$+\frac{1}{2\pi}\int_A^\infty [\bar{\tau}(x,c+iy)e^{(c+iy)t}+\bar{\tau}(x,c-iy)e^{(c-iy)t}]dy \tag{72}$$

where A is some large number which enables us to make some simplifications in the second integral. Recall that A_i and γ_i are functions of s, therefore functions of y. As an approximation we will take the limit as y goes to infinity of these quantities so that they may be taken outside of the integral, the remaining integral evaluated in closed form or else obtained from a tabulated result. When we do this we get the function

$$\bar{\tau}^*(x,s) = (d_0 e^{-st_2} + g_0 e^{-st_1})\frac{1}{s}$$

$$+ \sum_{i=1}^{3}[(d_{2i-1}e^{-st_2} + g_{2i-1}e^{-st_1})\frac{1}{s}\sinh\gamma_i^* x$$

$$+ (d_{2i}e^{-st_2} + g_{2i}e^{-st_1})\frac{1}{s}\cosh\gamma_i^* x] \qquad (73)$$

where the d_k $(k=0,\ldots,6)$ are determined by letting $T=T_0$ and the $g_k(k=0,\ldots,6)$ are determined by letting N_0, M_0, and Q_0 equal zero in equations (61-67). The $\gamma_i^*(i=1,2,3)$ are determined by letting $y\to\infty$ in equation 47. Note that d_k, g_k and γ_i are time independent. Now substitute this into (72). Also since $t_2<t_1$, let $t_2=0$.

$$\tau(x,t)= \frac{1}{2\pi}\int_0^A \bar{\tau}(x,c+iy)e^{(c+iy)t} + \bar{\tau}(x,c-iy)e^{(c-iy)t}dy$$

$$+ \frac{1}{2\pi}[d_0+\sum_{i=1}^{3} d_{2i-1}\sinh\gamma_i^* x+d_{2i}\cosh\gamma_i^* x]\int_A^\infty\{\frac{e^{(c+iy)t}}{c+iy} + \frac{e^{(c-iy)t}}{c-iy}\}dy$$

$$+ \frac{1}{2\pi}[g_0+\sum_{i=1}^{3} g_{2i-1}\sinh\gamma_i^* x+g_{2i}\cosh\gamma_i^* x]\int_A^\infty\{\frac{e^{(c+iy)(t-t_1)}}{c+iy}+\frac{e^{(c-iy)(t-t_1)}}{c-iy}\}dy.$$

$$(74)$$

Letting

$$D = d_0 + \sum_{i=1}^{3} d_{2i-1}\sinh\gamma_i^* x + d_{2i}\cosh\gamma_i^* x \qquad (75)$$

$$G = g_0 + \sum_{i=1}^{3} g_{2i-1}\sinh\gamma_i^* x + g_{2i}\cosh\gamma_i^* x \qquad (76)$$

we may write

$$\tau(x,t) = \frac{1}{2\pi}\int_A^\infty [\bar\tau(x,c+iy)e^{(c+iy)t} + \bar\tau(x,c-iy)e^{(c-iy)t}]dy$$

$$+ \frac{D}{2\pi}\int_A^\infty \frac{e^{ct}[2c\cos yt+2y\sin yt]}{c^2+y^2}\,dy$$

$$+ \frac{G}{2\pi}\int_A^\infty \frac{e^{c(t-t_1)}[2c\cos y(t-t_1)+2y\sin y(t-t_1)]}{c^2+y^2}\,dy. \qquad (77)$$

Letting

$$S_i(x) = \int_0^x \frac{\sin y}{y}\,dy \qquad (78)$$

and knowing

$$\int_0^\infty \frac{\sin y}{y}\,dy = \frac{\pi}{2}, \qquad (79)$$

we obtain

$$\tau(x,t)= \frac{1}{2\pi}\int_0^A [\bar\tau(x,c+iy)e^{(c+iy)t} + \bar\tau(x,c-iy)e^{(c-iy)t}]dy$$

$$+ \frac{D}{\pi}e^{ct}\left[\frac{c\cos At}{A} + \{\frac{\pi}{2} - Si(At)\}(1-ct)\right]$$

$$+ \frac{G}{\pi}e^{c(t-t_1)}\left[\frac{c\cos A(t-t_1)}{A} + (\frac{\pi}{2} - Si[A(t-t_1)])(1-c(t-t_1))\right]$$

$$(80)$$

where $\bar\tau$ is obtained from equation (49).

In a similar way we obtain

$$\sigma(x,t)= \frac{1}{2\pi}\int_0^A[\bar{\sigma}(x,c+iy)e^{(c+iy)t} + \bar{\sigma}(x,c-iy)e^{(c-iy)t}]dy$$

$$+ \frac{D^*}{\pi} e^{ct} [\frac{c \cos At}{A} + \{ \frac{\pi}{2} - Si(At)\}(1-ct)]$$

$$+ \frac{G^*}{\pi} e^{c(t-t_1)}[\frac{c \cos A(t-t_1)}{A} +(\frac{\pi}{2} - Si[A(t-t_1)])(1-c(t-t_1))] \tag{81}$$

where $\bar{\sigma}$ is obtained from equation (50).

And

$$D^* = \sum_{i=1}^{3} [d_{2i-1}(a_7^*\gamma_i^{*3}+a_8\gamma_i^*)\cosh\gamma_i^*x$$

$$+ d_{2i}(a_7^*\gamma_i^{*3}+a_8\gamma_i^*)\sinh\gamma_i^*x] \tag{82}$$

$$G^* = \sum_{i=1}^{3} [g_{2i-1}(a_7^*\gamma_i^{*3}+a_8\gamma_i^*)\cosh\gamma_i^*x$$

$$+ g_{2i}(a_7^*\gamma_i^{*3}+a_8\gamma_i^*)\sinh\gamma_i^*x] \tag{83}$$

$$a_7^* = \lim_{y\to\infty} a_7 \ . \tag{84}$$

4. The Numerical Integration

The integration in expressions 80 and 81 was performed using Simpson's Rule. Because of the oscillating nature of the integrand, such a scheme was found to be reliable. The choice of the upper limit of the integrand (A) is made according to convergence; increasing it until little change is seen in the result. A value of 80 was chosen. As reported in [11] the value of "A" necessary for good results was from 20 to 30. This was not the case in this study. The results were checked with an elastic solution for both small time and large time, and were found to be within 1% of these values.

In order to determine a numerical value of the integrand for some value of y, several steps must be performed. First the roots of the cubic equation (47) must be found. Note that the coefficients of the equation are complex numbers. A numerical scheme was used to find the roots. Then the 7 linear equations resulting from the boundary conditions (61-67) must be solved to obtain the constants A_0-A_6. Substituting these into equation (49) for a given value of x allows one to evaluate the integrand. Because of the calculations involved in equations (80) and (81), the integrand is a real number being made up of a pair of complex conjugates.

If "A" is chosen as 80 and the step length as .2 then the preceeding operation from beginning to end must be performed 400 times in order to calculate one value of stress. Because of the exponential e^{iyt}, for large values of time the oscillating nature of the integrand is emphasized and the integral is very difficult to evaluate. However, the solution reaches steady state before any numerical difficulties are encountered.

5. Results

The formulation presented here permits solution of a single lap joint or a cover plate under the combined loading of bending, tension, transverse shear, and temperature change (see figures 1a, b). A restriction is that when a change of temperature occurs, the adhesive must be stress free at t=0. Mechanical loads may be applied at any later time, i.e. $N_0(t) = NH(t-t_2)$ where $t_2>0$. Seven basic problems are considered as examples. They are a single lap joint in bending (figure 2c), in tension (figure 2d), and in transverse shear (figure 2e). The same separate loading conditions are considered for the cover plate (figures 2f-h). The seventh problem is that of temperature change. Since a plate theory is used, there is no difference between the thermal stress solution to a single lap joint or a cover plate because in both cases the boundary conditions are the same. In reality these are two different problems. The cover plate will have a symmetric solution, the single lap joint will not although it will probably be nearly symmetric. The solution obtained in this study is symmetric so it is more accurate to associate the thermal stress solution with the geometry of the cover plate.

These seven problems are solved for a fixed geometry so the solution to the general loading of either the cover plate or the single lap joint can be obtained by simple addition. The results for the adhesive shear and normal stresses are presented in tables (1-7). Also each of these separate problems is solved at four different operating temperatures, taking into account the functional dependence of the adhesive constants on the temperature. Therefore there are four solutions presented in each of these tables.

In addition to these results, tables (8-11) compare the solutions of the adhesive stresses for two different problems where one parameter has been varied or in tables (12,13) where the affect of transverse shear deformation in the plates has been investigated. Tables 8 and 9 show the affect that the bond length has on the solution for a single lap joint in bending. It is observed that the stresses near the bond edge are nearly independent of the bond length for values of ℓ within the restrictions of plate theory. This is not noticed in table 8 where stresses have been calculated at specific values of the non-dimensional variable x/ℓ. However, in table 9, the stresses are calculated at specific distances away from the left end using the variable x' where $x'=x+\ell$ and here the similarity is apparent. In this table the two values of ℓ are 20 mm and 100 mm. The results show the solution at the left end to be the same to three significant figures for about 11 mm.

The adhesive thickness is the only parameter that is different between the two solutions presented in table 10. The problem is a single lap joint subjected to bending. The results indicate that the thinner the adhesive layer, the higher the peak stresses at $x=\pm\ell$, shear stress being more affected than normal stress. This is probably because the normal stress is more uniform throughout the thickness than the shear stress which is actually confined to the upper and lower interface. It is the expected result.

In table 11 the thermal stress problem of a cover plate is considered. In 11a,c the upper plate is less "stiff" than the lower plate, while in 11b,d the relative stiffness is reversed. This is accomplished simply by varying the upper plate thickness. The peak normal stress changes from tension in 11c to compression in 11d. Shear changes very little. This situation is also illustrated in figures 4,5.

The affect of transverse shear deformation is investigated in tables 12 and 13. In table 12 the solution to the problem of a single lap joint in bending is presented for both Reissner plate theory and for classical plate theory. Table 13 similarly compares these two theories for a cover plate subject to temperature change. It was observed that for bending, extension, and for transverse shear loadinqs the peak shear stress was higher for Reissner theory while the peak normal stress was higher for classical theory. This is evident for bending in table 12. The opposite was true in the thermal stress problem (table 13).

In addition to the tables, there are also some figures showing basic trends and profiles. The distribution of the shear and the normal stress is presented in figures 6 and 7 respectively, for bending of a single lap joint. The shear stress is plotted for t=0 and t=1 hour, while the normal stress, which decays less, is only plotted for t=0. The time behavior of the peak stresses is shown in figure 8. Here it is evident that the shear stress, although lower than the normal stress, decays more. The only case where the peak shear stress was higher than the peak normal stress was the thermal stress result. This is shown in figure 9.

The material constants and dimensions used in the calculations are as follows:

Upper Plate: Graphite-Epoxy Plate

h_1 indicated on table or figure

1)[0 ± 45 90] laminated construction

Tables (1-7) , Figures (6-8)

$E_{x1} = 7.377 \times 10^{10}$ N/m^2

$E_{z1} = 4.826 \times 10^{10}$ N/m^2

$\mu_1 = 1.793 \times 10^{10}$ N/m^2

$\nu_{x1} = .29$

$\alpha_{x1} = 1.17 \times 10^{-6}$ °C^{-1}

$\alpha_{z1} = 3.6 \times 10^{-6}$ °C^{-1}

2) Unidirectionally oriented fibers

Tables (8-13) , Figures(4,5,9)

$E_{x1} = 1.448 \times 10^{11}$ N/m^2

$E_{z1} = 1.034 \times 10^{10}$ N/m^2

$\mu_1 = 4.482 \times 10^{9}$ N/m^2

$\nu_{x1} = .21$

$\alpha_{x1} = -4.5 \times 10^{-7}$ °C^{-1}

$\alpha_{z1} = 3.6 \times 10^{-5}$ °C^{-1}

Lower Plate: Aluminum used for all calculations

$h_2 = 2.286$ mm

$E_2 = 7.171 \times 10^{10}$ N/m^2

$\nu_2 = .33$

$\alpha_2 = 2.466 \times 10^{-5}$ °C^{-1}

Adhesive: typical epoxy

h_0 indicated on table or figure

ℓ indicated on table or figure

$$G(T,t) = \{[(\mu_0(T)-\mu_\infty(T)]e^{-t/\varepsilon(t)} + \mu_\infty(T)\}H(t) \tag{85}$$

the Laplace transform of this is needed for the numerical work

$$\bar{G}(T,s) = \frac{\mu_\infty(T)-\mu_\infty(T)}{s+1/\varepsilon(T)} + \frac{\mu_\infty(T)}{s} \tag{86}$$

where

$$\varepsilon(T) = \frac{\mu_\infty(T)}{\mu_0(T)} t_0(T) \tag{87}$$

$$\mu_0(T) = \lim_{t \to 0^+} G(T,t) \tag{88}$$

$$\mu_\infty(T) = \lim_{t \to \infty} G(T,t) \tag{89}$$

$t_0(T)$ is the retardation time

$$K(T) = \frac{E_0(T)\mu_0(T)}{3[3\mu_0(T)-E_0(T)]} \tag{90}$$

where the numerical values of the constants are as follows. These values are obtained from [11].

Table 14

T(°C)	$E_0(N/m^2)$	$\mu_0(N/m^2)$	$\mu_\infty(N/m\tau)$	t_0(hours)
21	3.206×10^9	1.241×10^9	5.516×10^8	.5
43	3.034×10^9	1.172×10^9	4.826×10^8	.5
60	2.827×10^9	1.089×10^9	3.999×10^8	.5
82	2.655×10^9	1.034×10^9	3.447×10^8	.5

6. Fracture of the Bond Edge, Formulation

In this section I will assume the adhesive to behave elastically. The only changes in the formulation will be in equations (16) and (18) which will be replaced by

$$\tau = G\gamma \tag{91}$$

$$\varepsilon_y = \frac{1-\nu-2\nu^2}{E(1-\nu)}\,\sigma_y - \frac{\nu}{1-\nu}\,\varepsilon_x \tag{92}$$

where the second relation is obtained from plane strain considerations.

From an energy balance of an elastic solid neglecting inertia forces we have

$$\frac{d}{dA}(U-V) = \gamma_F \tag{93}$$

where A is the crack area, U is the work done by external forces, V is the stored elastic energy, and γ_F is the fracture energy. If fixed grip conditions are assumed the work done by external forces is zero and (93) becomes

$$- \frac{dV}{dA} = \gamma_F \quad . \tag{94}$$

Consider a crack of length da to initiate at the bond edge. The volume enclosed by this portion of the adhesive is $\frac{1}{2} h_o dA$ for unit depth where h_o is the thickness of the bond and $dA = 2da$. Note that $\frac{2\, dV}{h_o dA}$ is then the stored energy per unit volume or simply the strain energy density function evaluated at the bond edge taking into account that stresses and strains have been averaged through the adhesive thickness and assuming that all stored energy is released upon deponding. Note that this assumes a tensile stress which tends to open the crack. For plane strain the strain energy density is given by

$$W = \frac{1}{2} [\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}] \quad . \tag{95}$$

Using Hooke's law to write W in terms of ε_x, σ_y and τ_{xy} we get:

$$W = \frac{1}{2} [\frac{E \varepsilon_x^{\,2}}{1-\nu^2} + \frac{\sigma_y^{\,2}}{E} \frac{1-\nu-2\nu^2}{1-\nu} + \frac{\tau^2_{\,xy}}{G}] \quad . \tag{96}$$

Since energy is being released, i.e. force and displacement are in opposite directions, $\frac{dV}{dA}$ is negative and (94) becomes

$$\gamma_F = \frac{h_o}{4} [\frac{E \varepsilon_x^{\,2}}{1-\nu^2} + \frac{\sigma_y^{\,2}}{E} \frac{1-\nu-2\nu^2}{1-\nu} + \frac{\tau^2_{\,xy}}{G}] \quad . \tag{97}$$

If a crack initiates while the bond edge is in compression then not all the energy will be released and the term in the strain energy density function corresponding to the normal stress should be ignored. For $\sigma_y<0$ we get

$$\gamma_F = \frac{h_0}{4}\left[\frac{E\varepsilon_x^{\,2}}{1-\nu^2} + \frac{\tau^2{}_{xy}}{G}\right] \quad . \tag{98}$$

It should be noted that in the preceeding analysis the treatment given to the shear stress is not very accurate. Actually the shear stress is zero on the free surface and infinite at the corners. The average value is used which may perhaps be significantly low when considering a crack growing from a corner. There is no way in the present analysis to correct for this.

7. Solution and Results

We want to calculate (96) at the end of the bond or say at $x=-\ell$. Therefore we need $\sigma_y(-\ell)$, $\tau_y(-\ell)$ and $\varepsilon_x(-\ell)$. The solution for τ and σ is already given. To determine $\varepsilon_x(-\ell)$ note the following.

From equation (8b)

$$\varepsilon_x = \left(\frac{du_1}{dx} - \frac{h_1}{2}\frac{d\beta_{1x}}{dx} + \frac{d_{u2}}{dx} + \frac{h_2}{2}\frac{d\beta_{2x}}{dx}\right)/2 \tag{99}$$

using (4a,b) with $T=T_0$ and (5a,b) we get

$$\varepsilon_x = [C_1N_{1x} - \frac{h_1}{2}D_1M_{1x} + C_2N_{2x} + \frac{h_2}{2}D_2M_{2x}]/2 \tag{100}$$

evaluating this at $x=-\ell$

$$\varepsilon_x(-\ell) = \frac{1}{2}[C_1 N_{1x}(-\ell) - \frac{h_1}{2} D_1 M_{1x}(-\ell) + C_2 N_{2x}(-\ell) + \frac{h_2}{2} D_2 M_{2x}(-\ell)] \tag{101}$$

where $N_{1x}(-\ell)$, $M_{1x}(-\ell)$, $N_{2x}(-\ell)$, $M_{2x}(-\ell)$ are given by the boundary conditions.

It may also be of interest to calculate the geometry of the "crack", i.e. the displacement (COD) and the rotation. To do this we need $v_1(-\ell)$, $v_2(-\ell)$, $\beta_1(-\ell)$, and $\beta_2(-\ell)$. To uniquely define the displacement field, values of u, v, and β must be specified at some point. I will choose

$$u_1(-\ell) = v_1(-\ell) = \beta_1(-\ell) = 0 \quad . \tag{102}$$

Recalling equations (8a) and (92) we may write

$$\varepsilon_y = \frac{1-\nu-2\nu^2}{E(1-\nu)}\sigma - \frac{\nu}{1-\nu}\varepsilon_x = (v_1-v_2)/h_0 \quad . \tag{103}$$

Now solving for v_2

$$v_2 = v_1 - \frac{h_0(1-\nu-2\nu^2)}{E(1-\nu)}\sigma + \frac{h_0\nu}{1-\nu}\varepsilon_x \tag{104}$$

now evaluating at $x=-\ell$, taking into account $v_1(-\ell) = 0$

$$v_2(-\ell) = -\frac{h_0(1-\nu-2\nu^2)}{E(1-\nu)}\sigma(-\ell) + \frac{h_0\nu}{1-\nu}\varepsilon_x(-\ell) \quad . \tag{105}$$

To determine $\beta_2(-\ell)$ recall equation (6b).

$$\beta_{2x} = \frac{Q_{2x}}{B_2} - \frac{dv_2}{dx} \quad . \tag{106}$$

Using (103) we get

$$\beta_{2x} = \frac{Q_{2x}}{B_2} - \{\frac{dv_1}{dx} - \frac{h_o(1-\nu-2\nu^2)}{E(1-\nu)} \frac{d\sigma}{dx} + \frac{h_o\nu}{1-\nu} \frac{d\epsilon_x}{dx}\} \quad . \tag{107}$$

From equation (6a) we can write

$$\frac{dv_1}{dx} = \frac{Q_{1x}}{B_1} - \beta_{1x} \quad . \tag{108}$$

The solution for the adhesive stresses in the case of an elastic adhesive can be found in [1]. They are given as

$$\tau(x) = K_o + \sum_{i=1}^{3} [K_{2i-1}\sinh\theta_i x + K_{2i}\cosh\theta_i x] \tag{109}$$

$$\sigma(x) = -\frac{1}{\alpha_2} \sum_{i=1}^{3} (\alpha_1\theta_i+\theta_i^3)(K_{2i-1}\cosh\theta_i x+K_{2i}\sinh\theta_i x) \tag{110}$$

where all constants are defined in [1]. The only difference between this solution and the one presented in this study is the substitution of equations (91) and (92) for (16) and (18). From (109) we obtain

$$\frac{d\sigma}{dx} = -\frac{1}{\alpha_2} \sum_{i=1}^{3} (\alpha_1+\theta_i^2)(K_{2i-1}\sinh\theta_i x+K_{2i}\cosh\theta_i x) \quad . \tag{111}$$

Using equation (100) with (1a,b) and (3a,b) we obtain

$$\frac{d\varepsilon_x}{dx} = \frac{1}{2}\{[C_1 + \frac{h_1}{2} D_1 \frac{h_1+h_o}{2} - \frac{h_2}{2} D_2 \frac{h_2+h_o}{2} - C_2]\tau$$

$$- \frac{h_1}{2} D_1Q_{1x} + \frac{h_2}{2} D_2Q_{2x}\} \quad . \tag{112}$$

Substituting (108), (111) and (112) into (107) and evaluating at $x=-\ell$, we obtain

$$\beta_{2x}(-\ell) = \frac{Q_2(-\ell)}{B_2} - \{ \frac{Q_1(-\ell)}{B_1} \quad \frac{h_o(1-\nu-2\nu^2)}{E(1-\nu)} \frac{d\sigma}{dx}\Big|_{x=-\ell}$$

$$+ \frac{h_o\nu}{1-\nu} \frac{d\varepsilon_x}{dx}\Big|_{x=-\ell} \} \tag{113}$$

where

$$\frac{d\sigma}{dx}\Big|_{x=-\ell} = \sum_{i=1}^{3} - \frac{1}{\alpha_2} (\alpha_1+\alpha_i{}^2)[-K_{2i-1}\sinh\theta_i\ell+K_{2i}\cosh\theta_i\ell] \tag{114}$$

$$\frac{d\varepsilon}{dx}\Big|_{x=-\ell} = \frac{1}{2}\{[C_1+ \frac{h_1}{2} D_1 \frac{h_1+h_o}{2} - \frac{h_2}{2} D_2 \frac{h_2+h_o}{2} - C_2]\tau(-\ell)$$

$$- \frac{h_1}{2} D_1Q_1(-\ell) + \frac{h_2}{2} D_2Q_2(-\ell)\} \quad . \tag{115}$$

Another interesting parameter from a fracture mechanics point of view is the stretch defined as

$\Delta = \dfrac{\delta-h_o}{h_o}$ where δ is the distance from one corner of the bond at adherend 1 to the other corner of the bond at adherend 2.

From simple kinematics

$$\delta = h_0\sqrt{(1+\varepsilon_y)^2 + \gamma_{xy}^2} \qquad (116)$$

$$\therefore\ \Delta = \sqrt{(1+\varepsilon_y)^2 + \gamma_{xy}^2} - 1\ , \qquad (117)$$

to get $\Delta(-\ell)$ we note that

$$\varepsilon_y(-\ell) = \frac{1}{h_0}\,(v_1(-\ell) - v_2(-\ell)) = -\,\frac{v_2(-\ell)}{h_0} \qquad (118)$$

$$\gamma_{xy}(-\ell) = \frac{\tau_{xy}(-\ell)}{G}\ . \qquad (119)$$

So

$$\Delta(-\ell) = \sqrt{\left(1 - \frac{v_2(-\ell)}{h_0}\right)^2 + \frac{\tau^2}{G}} - 1\ . \qquad (120)$$

The following example was considered for some brief calculations.

Upper and Lower plate: Aluminum

$E = 7.239\text{x}10^{10}$ N/m^2

$\nu = .33$.

Adhesive

$E = 1.931\text{x}10^9$ N/m^2

$\nu = .40$

$h_0 = .127$ mm .

Loading: $N_0 = 1.112\text{x}10^4$ N .

The problem considered was the extension of a cover plate (see figure 2g). Values of ℓ were varied from 25.4 mm to 254 mm. It was found, like the results of tables 8 and 9, that the results were not dependent on ℓ. The results are

case a) $h_1 = 3.175\text{mm}$, $h_2 = 3.175\text{mm}$

$\gamma_F = 36.92 \text{ N}\cdot\text{m/m}^2$

$v_2(-\ell) = 7.562 \times 10^{-4}$ mm

$\beta_2(-\ell) = 9.705 \times 10^{-5}$

$\Delta(-\ell) = 12.02$.

case b) $h_1 = 6.35$ mm, $h_2 = 3.175$ mm

$\gamma_F = 22.6 \text{ N}\cdot\text{m/m}^2$

$v_2(-\ell) = 7.184 \times 10^{-5}$ mm

$\beta_2(-\ell) \sim 0$

$\Delta(-\ell) = 9.193$.

Recall (102) where the assumption was made that $u_1(-\ell) = v_1(-\ell) = \beta_1(-\ell) = 0$.

It should be noted that in case b, the bond edge is in compression and that the fracture energy is calculated using equation (93). In case a the normal stress is very nearly zero.

Part II

Heat Generation of a Viscoelastic Material

1. Introduction

Because of the viscoelastic nature of the adhesive and perhaps also of the adherends, temperature considerations are important in the design of a bonded joint. Not only do material properties change with changing temperatures (treated in Part I), but temperature increases may occur due to viscous dissipation incurred during loading, especially cyclic loading. This phenomenon is illustrated in a test done by Nasa (see figure 10) where at intervals of 10,000 cycles the displacement of a cycling specimen is recorded versus time. One observes an increase in the net displacement and also of the displacement amplitude. Since the loading stays the same, as seen on the lower portion of the graph, the only explanation here is that material properties change. One parameter that is not recorded in these experiments is temperature, but this is known to go up due to viscous dissipation as seen from experiments done by the author. The conclusion is that the dependence of material property behavior or temperature may be causing the increasing displacement amplitude. The change in net displacement can be attributed to both temperature change and to creep.

From the behavior shown in figure 10, it is evident that temperature effects are important in design when cyclic loading of viscous materials exists, a case of which the bonded joint is a good example. However, the incorporation of these considerations into the analysis of the bonded joint is rather difficult and therefore will be treated separately in this section.

The problem investigated, both theoretically and experimentally, consists of a one-dimensional specimen subjected to a cyclic loading at t=0 (see figures11a,b). In the theory the temperature is predicted, in the experiment the temperature is recorded. The results are then compared. Again, because of analytical and experimental difficulties, the theory does not take into account the temperature dependence of material properties. This limits the solution to temperature ranges over which these changes are small. The theory also neglects inertia forces, the effect of which is believed to be small for frequencies considered in this study. In the solution of the heat equation the coupling term is included, but its effect is shown to be negligible.

2. Experimental Work

Experiments performed in this study were simple. A plexiglas specimen (figure 11a) was cycled in tension on an MTS machine at varying frequencies. Temperature measurements were taken by use of a thermocouple attached at the center of the specimen and connected to a digital thermometer. A small hole was drilled in the center of the specimen to accommodate the thermocouple. The specimen was insulated by cotton wrapped in aluminum foil. Reinforcement of the specimen was necessary at the ends, which was accomplished by bonding plexiglas plates of the same thickness using a solvent cement marketed as IPS Weld-On 4.

The loading was sinusoidal varying from 1.103×10^7 N/m^2 to 3.309×10^7 N/m^2. The upper load level is approximately 40% of the failure load. There was some problem with fatigue cracks emanating from the drilled hole. This ended the test of the 50 hertz specimen, which appeared to be headed for a range of possible melting. The glass transition temperature for plexiglas is about 72°C. Theoretical results indicate 79°C as an asymptote.

The recording of the displacement history of the specimen was not possible at frequencies above about 3 hertz because of the instruments used. Therefore records like those of figure 10 obtained by Nasa were not possible.

3. Analytical Modeling, Formulation and Solution

An explanation of the phenomenon of rising temperature in a specimen under cyclic loading is straightforward. As the specimen is subjected to load, accompanying strain causes internal viscous action which generates heat. As one observes the load-displacement curve through one cycle, a hysteresis loop shows that there is energy loss equal to the area enclosed. Several of these loops are shown in figure 12 for varying frequencies. In this study all energy loss was assumed to go directly into heat. Perhaps some of this energy was expended or used in some other form which may relate to the microstructural changes in the material, but this was not taken into account. Perhaps the percentage of dissipated energy that goes into heat can be taken as a variable, or could indeed be determined as being an unknown.

From this basis, for any theoretical study, one needs to know the displacements in the material under given loads. Therefore a model must be chosen that describes the constitutive relations for the material. For this purpose, a spring-dashpot assembly is chosen as shown in figure 13a.

The problem now consists of three parts. First, a material characterization must be made. This involves the fitting of an experimentally obtained creep curve (see figure 14) to the curve defined by the above chosen constitutive law. The second is the calculation of the heat input that goes into the energy equation. The solution of this equation is the third and final step.

The form of the creep curve is given by the creep compliance $J(t)$ where

$$J(t) = \frac{\varepsilon(t)}{\sigma_0} \quad . \tag{121}$$

$\varepsilon(t)$ is the strain resulting from the loading $\sigma_0 H(t)$, where $H(t)$ is the unit step function. Using the general model shown in figure 13a we obtain

$$J(t) = \frac{1}{E} + \frac{t}{\lambda} + \sum_{i=1}^{N} \frac{1}{E_i} \left(1 - e^{-\frac{E_i}{\lambda_i} t}\right) \quad . \tag{122}$$

The creep curve shown in figure 14 is fit to the model shown in figure 13b. The numerical values of the constants are also given in this figure. The curve fitting procedure is outlined in appendix A . A comparison of the two curves is shown in table 15.

It should be noted here that in recording a creep curve experimentally there are difficulties for small time, i.e. starting the test. Theoretically the loading is given by $\sigma(t) = \sigma_0 H(t)$ which experimentally is impossible to apply (see the creep curve, figure 14). An accurate description of the creeping phenomenon for $t < 2$ seconds is important as it has a great influence on the results of the analysis. With the given creep curve this small time behavior was approximated as follows.

In the creep test (figure 14) the data were read directly from the graph. The problem was that it took about 4 seconds to increase the load to σ_0(3.307×10^7 N/m^2) and during this time there was significant creeping. It is, therefore, difficult to determine the initial elastic response which appears to be about 6.4 units on the graph. In the next 4 seconds the specimen creeps about 0.2 units. It was approximated that during the first 4 seconds the displacement due to creep would have been about 0.2 units. Since the average load during the first 4 seconds is half of σ_0, I estimated the actual creep to be 0.1 unit and that the elastic response was actually 6.3 units. This is how the values in table 15 are obtained.

A possible improvement to this complication would be to calculate the response to the loading $\sigma(t) = \sigma_0 t\ H(t)$, (a ramp load). This can be applied accurately in an experiment. For the form of this curve see appendix B. Note that this method assumes a linear material behavior. In either method the main problem is the determination of the initial elastic constants.

Another problem often encountered in representing a creep curve deals with the other extreme of the time scale, the large time behavior. Usually a creep test is not run long enough to accurately determine the asymptotic slope of this curve. For a solid the curve will have zero slope or in terms of the model of figure 13a, infinite λ. A positive slope is characteristic of a material with fluid behavior. In the problem considered in this study it was found that the results were not sensitive to possible values of λ and that the assumption that plexiglas was a solid was sufficient.

Given the creep compliance, with the use of the hereditary integrals, one can find the strain for any loading. The derivation of this relation can be found in Flugge [13].

$$\varepsilon(t) = \sigma(t)\, J(0) + \int_0^t \sigma(t) \frac{dJ(t-t')}{d(t-t')}\, dt' \; . \tag{123}$$

The loading in the experiments is given by

$$\sigma(t) = d + e \sin\omega t \; . \tag{124}$$

Substitution of this into (123) gives

$$\varepsilon(t) = \frac{d}{E} + \frac{e}{E} \sin\omega t + \frac{d}{\lambda} t - \frac{e}{\lambda\omega}(\cos\omega t - 1) + \sum_{i=1}^{N} \frac{d}{E_i}\left(1 - e^{-\frac{E_i}{\lambda_i} t}\right)$$

$$+ \sum_{i=1}^{N} \frac{eE_i}{E_i^2+\omega^2\lambda_i^2} \sin\omega t - \sum_{i=1}^{N} \frac{\omega_e\lambda_i}{E_i^2+\omega^2\lambda_i^2} \cos\omega t$$

$$+ \sum_{i=1}^{N} \frac{e\lambda_i\omega}{E_i^2+\omega^2\lambda_i^2}\, e^{-\frac{E_i}{\lambda_i} t} \; . \tag{125}$$

An alternate technique for determination of $\varepsilon(t)$ is given in appendix C.

Before proceding with the derivation let us look at the one-dimensional energy equation as found in Boley and Weiner [14].

$$\sum_{i=1}^{N} \sigma_{\lambda i} \dot{\varepsilon}_{\lambda i} + \sigma_{\lambda} \dot{\varepsilon} + k \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} + 9K(t)\alpha T_0 \dot{\varepsilon}(t) , \qquad (126)$$

where the subscript λ means the stress or strain that is in the dashpot. $K(t)$ is the bulk modulus, α is the thermal coefficient of expansion, and T_0 is the reference temperature. The last term in equation (126) is the coupling term. If we neglect this term, equation (126) has the following more familiar form

$$Q(t) + k \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} , \qquad (127)$$

where $Q(t)$ is the heat generation term or energy per unit volume per unit time. It may also be thought of as the rate that work is done per unit volume. The work done per unit volume is

$$\int \sigma_{\lambda}(t)\dot{\varepsilon}_{\lambda}(t)dt \quad . \qquad (128)$$

Again the subscript λ is used because the work done in the spring does not contribute to heating.

The rate at which work is done is

$$Q_0(t) = \frac{d}{dt} \int \sigma_{\lambda}(t)\dot{\varepsilon}_{\lambda}(t)dt \quad . \qquad (129)$$

If this is differentiated we get the terms in equation (126).

If we neglect the variation over one cycle and use an average value we obtain

$$Q_0(t) = \frac{1}{T}\int_{nT}^{(n+1)T} \sigma(t)\dot{\varepsilon}(t)dt \quad , \tag{130}$$

where T is the period and n refers to the n^{th} cycle. The subscript λ may now be dropped because the integral calculates the loss through the n^{th} cycle and any elastic contribution will integrate out to zero.

Performing this integration and letting $t = \frac{2\pi n}{\omega}$ we obtain:

$$Q_0(t) = \frac{1}{2}\left[\frac{e^2}{\lambda} + e\sum_{i=1}^{N} C_i\right] + \frac{d^2}{\lambda}$$

$$+ \sum_{i=1}^{N} a_i \left[\frac{e\lambda_i^2\omega}{E_i^2+\omega^2\lambda_i^2} + \frac{d\lambda_i}{E_i}\right]\left[e^{-\frac{E_i}{\lambda_i}} - e^{\frac{E_i}{\lambda_i}\left(t+\frac{2\pi}{\omega}\right)}\right]\frac{\omega}{2\pi} \,, \tag{131}$$

where

$$a_i = -\frac{E_i}{\lambda_i}\left[\frac{e\lambda_i\omega}{\omega^2\lambda_i^2+E_i^2} - \frac{d}{E_i}\right] \quad C_i = \frac{e\lambda_i\omega^2}{E_i^2+\lambda_i^2\omega^2} \quad . \tag{132}$$

The solution of equation (127) with the heat generation Q as given by (131) has the form: (see appendix D for solution technique and boundary conditions)

$$T(x,t) = T_0 - A\left(\frac{x^2}{2a} - \frac{\ell^2}{8a}\right) + \sum_{j=1}^{\infty} \frac{4A(-1)^j \ell^2}{(2j-1)^3 \pi^3 a} \cos \frac{(2j-1)\pi x}{\ell} e^{s_j t}$$

$$+ \sum_{i=1}^{N} \{ B_i \frac{e^{b_i t}}{b_i} [1 - \frac{\cosh \sqrt{\frac{b_i}{a}}\, x}{\cosh \sqrt{\frac{b_i}{a}}\, \frac{\ell}{2}}]\}$$

$$+ \sum_{i=1}^{N} \{ \sum_{j=1}^{\infty} \frac{4B(-1)^j \ell^2}{(2j-1)\pi[(2j-1)^2\pi^2 a + b_i \ell^2]} \cos \frac{(2j-1)\pi x}{\ell} e^{s_j t} \},$$

(133)

where

$$s_j = - \frac{(2j-1)^2 \pi^2 a}{\ell^2} \tag{134}$$

$$A = [\frac{1}{2} \frac{e^2}{\lambda} + \frac{d^2}{\lambda} + \frac{e^2}{\lambda} \sum_{i=1}^{N} \frac{\lambda_i \omega^2}{\lambda_i^2 \omega^2 + E_i^2}]/\rho c \tag{135}$$

$$B_i = \frac{\omega}{2\pi\rho c} [(\frac{e\lambda_i \omega}{E_i^2 + \omega^2 \lambda_i^2})^2 - (\frac{d}{E_i})^2] E_i (e^{-\frac{E_i}{\lambda_i}\frac{2\pi}{\omega}} - 1) \tag{136}$$

$$b_i = - \frac{E_i}{\lambda_i} \tag{138}$$

$$a = \frac{k}{\rho c} \quad . \tag{138}$$

If the coupling term is included in equation (2), there is no point in time-averaging the heat generation, as there is no extra work involved in taking it as it is. Here an assumption is made regarding the bulk modulus K. As in the bonded joint problem, K is assumed to be time independent. Substituting everything into (2) and using relations (C2) and (C5) of appendix C we find

$$\rho c \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = \sum_{i=1}^{N} \lambda_i A_i{}^2 e^{2\alpha_i t} + \sum_{i=1}^{N} \lambda_i B_i{}^2 \cos^2\omega t + \sum_{i=1}^{N} \lambda_i C_i{}^2 \sin^2\omega t$$

$$+ \sum_{i=1}^{N} \lambda_i 2A_i B_i \cos\omega t e^{\alpha_i t} + \sum_{i=1}^{N} \lambda_i 2A_i C_i \sin\omega t e^{\alpha_i t}$$

$$+ \sum_{i=1}^{N} \lambda_i 2B_i C_i \sin\omega t \cos\omega t + \frac{d^2}{\lambda} + \frac{2de}{\lambda} \sin\omega t + \frac{e^2}{\lambda} \sin^2\omega t$$

$$- 9K\alpha T_0 \frac{d}{\lambda} - 9K\alpha T_0 \left(\frac{e}{\lambda} + \sum_{i=1}^{N} C_i\right) \sin\omega t$$

$$- 9K\alpha T_0 \left(\frac{e\omega}{E} + \sum_{i=1}^{N} B_i\right) \cos\omega t - 9K\alpha T_0 \sum_{i=1}^{N} A_i e^{\alpha_i t}, \qquad (139)$$

where

$$A_i = - \frac{E_i}{\lambda_i} \left[\frac{e\lambda_i\omega}{\lambda_i{}^2\omega^2 + E_i{}^2} - \frac{d}{E_i} \right] \qquad (140)$$

$$B_i = \frac{E_i e\omega}{\lambda_i{}^2\omega^2 + E_i{}^2} \qquad (141)$$

$$C_i = \frac{e\lambda_i\omega^2}{\lambda_i{}^2\omega^2 + E_i{}^2} \qquad (142)$$

$$\alpha_i = -\frac{E_i}{\lambda_i} \quad . \tag{143}$$

After expressing time dependent quantities in exponential form using complex variables, and after defining more constants, we obtain

$$\rho c \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = [A + \frac{E}{2} + \frac{F}{2}] + e^{i\omega t}[\frac{B}{2i} + \frac{C}{2}] + e^{-i\omega t}[\frac{-B}{2i} + \frac{C}{2}]$$

$$+ e^{2i\omega t}[\frac{E}{4} - \frac{F}{4} + \frac{I}{4i}] + e^{-2i\omega t}[\frac{E}{4} - \frac{F}{4} - \frac{I}{4i}] + \sum_{j=1}^{N} D_j e^{2\alpha_j t}$$

$$+ \sum_{j=1}^{N} J_j e^{\alpha_j t} + \sum_{j=1}^{N} (\frac{G_j}{2} + \frac{H_j}{2i}) e^{(\alpha_j + i\omega)t}$$

$$+ \sum_{j=1}^{N} (\frac{G_j}{2} - \frac{H_j}{2i}) e^{(\alpha_j - i\omega)t} \quad , \tag{144}$$

where

$$A = \frac{d^2}{\lambda} - 9K\alpha T_0 \frac{d}{\lambda} \tag{145}$$

$$B = \frac{2de}{\lambda} - 9K\alpha T_0(\frac{e}{\lambda} + \sum_{i=1}^{N} C_i) \tag{146}$$

$$C = -9K\alpha T_0(\frac{e\omega}{E} + \sum_{i=1}^{N} B_i) \tag{147}$$

$$E = \sum_{i=1}^{N} \lambda_i B_i{}^2 \tag{148}$$

$$F = \sum_{i=1}^{N} \lambda_i C_i{}^2 + \frac{e^2}{\lambda} \tag{149}$$

$$I = \sum_{i=1}^{N} \lambda_i 2B_i C_i \tag{150}$$

$$D_i = \lambda_i A_i{}^2 \tag{151}$$

$$G_i = 2\lambda_i A_i B_i \tag{152}$$

$$H_i = 2\lambda_i A_i C_i \tag{153}$$

$$J_i = -9K\alpha T_0 A_i \quad , \tag{154}$$

or

$$\frac{\partial T}{\partial t} - \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2} = \gamma_0 + \sum_{i=1}^{4+4N} \gamma_i e^{\beta_i t} \quad , \tag{155}$$

where

$$\gamma_0 = [A + \frac{E}{2} + \frac{F}{2}]/\rho c \tag{156}$$

$$\gamma_1 = (\frac{B}{2i} + \frac{C}{2})/\rho c \qquad \beta_1 = i\omega \tag{157}$$

$$\gamma_2 = (\frac{-B}{2i} + \frac{C}{2})/\rho c \qquad \beta_2 = -i\omega \tag{158}$$

$$\gamma_3 = \left(\frac{E}{4} - \frac{F}{4} + \frac{I}{4i}\right)/\rho c \qquad \beta_3 = 2i\omega \tag{159}$$

$$\gamma_4 = \left(\frac{E}{4} - \frac{F}{4} - \frac{I}{4i}\right)/\rho c \qquad \beta_4 = -2i\omega \tag{160}$$

for $j = 5,\ 5+N-1$

$$\gamma_j = [D_{j-4}]/\rho c \qquad \beta_j = 2\alpha_{j-4} \tag{161}$$

for $j = 5+N,\ 5+2N-1$

$$\gamma_j = (J_{j-N-4})/\rho c \qquad \beta_j = \alpha_{j-N-4} \tag{162}$$

for $j = 5+2N,\ 5+3N-1$

$$\gamma_j = \left(\frac{1}{2} G_{j-2N-4} + \frac{1}{2i} H_{j-2N-4}\right)/\rho c \qquad \beta_j = \alpha_{j-2N-4} + i\omega \tag{163}$$

for $j = 5+3N,\ 5+4N-1$

$$\gamma_j = \left[\frac{1}{2} G_{j-3N-4} - \frac{1}{2i} H_{j-3N-4}\right]/\rho c \qquad \beta_j = \alpha_{j-3N-4} - i\omega. \tag{164}$$

Using the solution in Appendix D we find

$$T(x,t) = T_0 - \gamma_0\left[\frac{x^2}{2a} - \frac{\ell^2}{8a}\right] + \sum_{n=1}^{\infty} \frac{4\gamma_0(-1)^n \ell^2}{(2n-1)^3\pi^3 a} \cos\frac{(2n-1)\pi x}{\ell}\, e^{s_n t}$$

$$+ \sum_{i=1}^{4+4N} \gamma_i \frac{e^{\beta_i t}}{\beta_i}\left[1 - \frac{\cosh\sqrt{\frac{\beta_i}{a}}\,x}{\cosh\sqrt{\frac{\beta_i}{a}}\,\frac{\ell}{2}}\right]$$

$$+ \sum_{i=1}^{4+4N} \sum_{n=1}^{\infty} \frac{4\gamma_i(-1)^n \ell^2}{(2n-1)\pi[(2n-1)^2\pi^2 a + \beta_i \ell^2]} \cos\frac{(2n-1)\pi x}{\ell}\, e^{s_n t}. \qquad (165)$$

Evaluating this expression at x=0, we obtain the form of the expression used for the results.

$$T(0,t) = T_0 + \gamma_0 \frac{\ell^2}{8a} + \sum_{n=1}^{\infty} \frac{4\gamma_0(-1)^n \ell^2}{(2n-1)^3\pi^3 a}\, e^{s_n t}$$

$$+ \sum_{i=1}^{4+4N} \gamma_i \frac{e^{\beta_i t}}{\beta_i}\left[1 - \frac{1}{\cosh\sqrt{\frac{\beta_i}{a}}\,\frac{\ell}{2}}\right]$$

$$+ \sum_{i=1}^{4+4N} \sum_{n=1}^{\infty} \frac{4\gamma_i(-1)^n \ell^2}{(2n-1)\pi[(2n-1)^2\pi^2 a + \beta_i \ell^2]}\, e^{s_n t} \quad . \qquad (166)$$

4. Discussion of Results

Before a comparison can be made between theoretical and experimental results, it is necessary to look at the theoretical modeling of the experimental results and to justify the choice of the parameters used in the theory. The analytical solution is based on the following assumptions: 1) heat is generated evenly throughout the domain, 2) the domain is one-dimensional, 3) the ends of the domain are held at constant temperature for all time, 4) the specimen is insulated along its length.

It is not possible to satisfy all of these points because there is no well defined length parameter in the experiment. The geometry of the specimen (figures 11a,c) shows that in order to satisfy "1" and "2" a length of 25.4 mm or $2\ell=25.4$ mm, should be used. If ℓ is chosen larger than this, the width of the specimen is not constant and therefore the heat generation, which is inversely proportional to the square of the width, is not uniform. (This inverse relationship can be seen from equation (131) taking into account the inverse dependence of stress on width). The boundary condition $T(\pm\ell,t) = T_{initial}$ (i.e., the assumption 3), is not satisfied for $2\ell = 25.4$ mm but despite this, this value of ℓ was chosen for the analytical solution. The affect of this on the comparison of solutions should be for the predicted temperature to be lower than the experimental value due to heat being conducted out more readily. One compensation here is that the insulation in the experiment is not perfect, as assumed in the theory, and therefore escaping heat in the experiment would tend to bring the two curves closer together.

The other remaining parameters to be defined are material properties. Besides the creep curve constants shown in figure 13b, numerical values chosen for the thermomechanical constants of Plexiglas (polymethylmethacrylate) are:

Bulk Modulus $K = 2.382 \times 10^9$ N/m^2

Thermal Diffusivity $a = \frac{k}{\rho c} = .001276$ cm^2 sec^{-1}

Thermal Conductivity $k = .00154$ Watt cm^{-1}°C^{-1}

Coefficient of Thermal Expansion $\alpha = .00009$°C^{-1} .

The thermal properties were obtained from [15]. It was assumed that the bulk modulus was time independent. This made it analytically possible to include the coupling term.

It should be mentioned that the fourth component in the spring-dashpot model (figure 13b) contributes almost all of the generated heat. This is the component that describes large creeping initially. An accurate determination of its constants E_4 and λ_4 depends on accurate small time creep readings, which are hard to obtain as previously discussed. The main difficulty appears to be in separating the initial elastic response from the small time creep behavior.

The theoretical results compare very well with the experimental curves (see figures 15, 16, 17, 18). Because of the discrepency in the boundary condition $T(\pm\ell,t) = T_{initial}$, the curves were not expected to be so close. One factor that has a great influence on the comparison is the choice of the thermal constants. There was a range reported in the literature created by the work of two or three researchers. It is possible that more favorable constants could have been used. For example, a larger value of the thermal conductivity would have lowered the asymptote in the theoretical curves. Perhaps the most impressive part of the solution is the functional dependence on ω, shown separately in figure 19 where temperature is plotted as a function of time (19a) and number of cycles (19b). Here it should be noted that for large values of ω where there are great changes in temperature, the solution becomes less valid because the material properties were taken to be temperature independent. Initially, however, the solution is valid for any frequency until inertia forces become important. The frequency level at which such effects must be taken into consideration may be approximated by the natural frequency of the material which is much higher than values in this study.

A comparison was made between the three different theories used in solving the heat equation. The simplest theory time-averaged the heat generation per cycle (equation (126) using (131) as the heat input). The solution was also obtained for the actual heat generation (equation (126) without coupling), and a third solution included the coupling term (equation 126). It was found that time averaging the heat generation per cycle is sufficient for the temperature profile (see tables 16, 17). If the details of the temperature are desired through a single cycle then one must include the coupling term but this effect seems to be rather insignificant (see tables 18, 19). Boley and Weiner note that a solution like the one obtained here involving thermoelastic dissipation, is meaningless without the inclusion of the coupling term. For the specific example solved here, this proved not to be true [14].

5. Conclusions

From the experiments performed, it is evident that temperature rise due to viscous dissipation is a significant factor in design. The analytical modeling of this phenomena, although not perfect because of the difficulties with the small time creep curve, has proved to work reasonably well. It shows for one thing that time-averaging the loss over one cycle is sufficient.

The small time creep curve can actually be obtained from the results of the temperature curve. All that is needed is the initial slope of the temperature curve. The relationship for small time is

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{2}\frac{e^2}{\lambda} + \frac{d^2}{\lambda} + \frac{1}{2} e \sum_{i=1}^{N} \frac{e\lambda_i \omega^2}{E_i^2 + \lambda_i^2 \omega^2} \quad . \tag{167}$$

For the example solved in this study, the fourth component of the spring-dashpot dominates the right hand side of this expression. As a further approximation we can write

$$\rho c \frac{\partial T}{\partial t} \cong \frac{1}{2} \frac{e^2 \lambda_4 \omega^2}{E_4{}^2 + \lambda_4{}^2 \omega^2} \quad . \tag{168}$$

If two curves are available for two different frequencies, a good guess for E_4 and λ_4 can be obtained by the above formula.

Because of the dominance of the fourth component of the model, it was also found that the argument whether the material is a solid ($\lambda \to \infty$) or a fluid ($\lambda < \infty$) is unimportant. In many cases the creep curve can not be run long enough to see if the curve reaches an asymptote in which case the material is a solid. It has been found that the value of λ does not influence the temperature profile too much and therefore a creep test need not be run for a long time.

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.651E+03	.579E+03	.455E+03	.384E+03	.377E+03
-.98	.417E+03	.381E+03	.316E+03	.273E+03	.268E+03
-.94	.124E+03	.126E+03	.124E+03	.117E+03	.115E+03
-.90	.517E+01	.160E+02	.331E+02	.401E+02	.397E+02
-.80	-.251E+02	-.197E+02	-.891E+01	-.998E+00	-.128E+00
-.70	-.951E+01	-.790E+01	-.413E+01	-.304E+00	.447E+00
-.60	-.344E+01	-.301E+01	-.174E+01	.692E-01	.606E+00
-.40	-.524E+00	-.527E+00	-.425E+00	-.559E-01	.161E+00
-.20	-.834E-01	-.974E-01	-.111E+00	-.622E-01	-.302E-02
0.00	-.288E-01	-.436E-01	-.848E-01	-.146E+00	-.168E+00
.20	-.108E+00	-.163E+00	-.317E+00	-.557E+00	-.664E+00
.40	-.691E+00	-.928E+00	-.151E+01	-.223E+01	-.246E+01
.60	-.449E+01	-.534E+01	-.715E+01	-.879E+01	-.910E+01
.70	-.115E+02	-.128E+02	-.154E+02	-.173E+02	-.175E+02
.80	-.290E+02	-.304E+02	-.327E+02	-.337E+02	-.336E+02
.90	-.679E+02	-.671E+02	-.648E+02	-.621E+02	-.614E+02
.94	-.917E+02	-.887E+02	-.824E+02	-.771E+02	-.761E+02
.98	-.124E+03	-.117E+03	-.104E+03	-.950E+02	-.937E+02
1.00	-.148E+03	-.137E+03	-.118E+03	-.106E+03	-.105E+03
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	-.206E+04	-.202E+04	-.195E+04	-.192E+04	-.192E+04
-.98	-.686E+03	-.692E+03	-.701E+03	-.708E+03	-.709E+03
-.94	.250E+03	.241E+03	.224E+03	.214E+03	.213E+03
-.90	.291E+03	.293E+03	.297E+03	.300E+03	.300E+03
-.80	.490E+02	.507E+02	.544E+02	.576E+02	.581E+02
-.70	.336E+01	.262E+01	.128E+01	.512E+00	.505E+00
-.60	.729E+00	.396E+00	-.312E+00	-.912E+00	-.991E+00
-.40	.170E+00	.155E+00	.993E-01	-.290E-03	-.344E-01
-.20	.252E-01	.258E-01	.203E-01	-.380E-02	-.198E-01
0.00	-.125E-02	-.343E-02	-.114E-01	-.290E-01	-.402E-01
.20	-.336E-01	-.468E-01	-.804E-01	-.124E+00	-.140E+00
.40	-.222E+00	-.274E+00	-.388E+00	-.494E+00	-.513E+00
.60	-.143E+01	-.155E+01	-.176E+01	-.186E+01	-.185E+01
.70	-.387E+01	-.395E+01	-.402E+01	-.393E+01	-.387E+01
.80	-.126E+02	-.124E+02	-.118E+02	-.112E+02	-.110E+02
.90	-.298E+02	-.280E+02	-.248E+02	-.228E+02	-.226E+02
.94	-.106E+02	-.857E+01	-.529E+01	-.383E+01	-.382E+01
.98	.955E+02	.923E+02	.861E+02	.816E+02	.809E+02
1.00	.221E+03	.209E+03	.188E+03	.175E+03	.174E+03

Table 1. Adhesive stresses for a single lap joint subjected to bending ($M_0 \neq 0$, $Q_0=N_0=\Delta T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=2.54\times10^{-2}$m.

T=43°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.628E+03	.549E+03	.418E+03	.350E+03	.344E+03
-.98	.406E+03	.366E+03	.295E+03	.253E+03	.248E+03
-.94	.125E+03	.126E+03	.123E+03	.114E+03	.112E+03
-.90	.883E+01	.204E+02	.376E+02	.432E+02	.425E+02
-.80	-.237E+02	-.174E+02	-.551E+01	.248E+01	.313E+01
-.70	-.912E+01	-.716E+01	-.269E+01	.154E+01	.223E+01
-.60	-.334F+01	-.276E+01	-.112E+01	.105E+01	.160E+01
-.40	-.532E+00	-.517E+00	-.332E+00	.197E+00	.460E+00
-.20	-.891E-01	-.103E+00	-.106E+00	-.161E-01	.664E-01
0.00	-.338E-01	-.533E-01	-.107E+00	-.185E+00	-.210E+00
.20	-.126F+00	-.198E+00	-.402E+00	-.716E+00	-.843E+00
.40	-.770E+00	-.107E+01	-.179F+01	-.266E+01	-.291E+01
.60	-.478F+01	-.578E+01	-.787E+01	-.966E+01	-.995E+01
.70	-.120F+02	-.135E+02	-.163E+02	-.182E+02	-.184E+02
.80	-.295E+02	-.310E+02	-.333E+02	-.341E+02	-.339E+02
.90	-.676F+02	-.665E+02	-.637E+02	-.606E+02	-.599E+02
.94	-.906E+02	-.871E+02	-.800E+02	-.743E+02	-.732E+02
.98	-.122E+03	-.114E+03	-.996F+02	-.903E+02	-.890E+02
1.00	-.144E+03	-.132E+03	-.112E+03	-.100E+03	-.989E+02
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	-.202E+04	-.197E+04	-.190E+04	-.187E+04	-.187E+04
-.98	-.686E+03	-.692E+03	-.701E+03	-.707E+03	-.708E+03
-.94	.241E+03	.230E+03	.211E+03	.201E+03	.201F+03
-.90	.290F+03	.292E+03	.296E+03	.298E+03	.298E+03
-.80	.505E+02	.527E+02	.571E+02	.606E+02	.611E+02
-.70	.319E+01	.237E+01	.976F+00	.320E+00	.351E+00
-.60	.607E+00	.207E+00	-.615E+00	-.125E+01	-.131E+01
-.40	.167E+00	.145E+00	.678F-01	-.566E-01	-.917E-01
-.20	.258E-01	.253E-01	.145E-01	-.214E-01	-.413E-01
0.00	-.190E-02	-.507E-02	-.167E-01	-.420E-01	-.565E-01
.20	-.379E-01	-.546E-01	-.970F-01	-.151E+00	-.168E+00
.40	-.240E+00	-.302E+00	-.434E+00	-.549E+00	-.566E+00
.60	-.147E+01	-.160E+01	-.182E+01	-.190E+01	-.188E+01
.70	-.391E+01	-.399E+01	-.403E+01	-.388E+01	-.381E+01
.80	-.126E+02	-.123E+02	-.117E+02	-.109E+02	-.107E+02
.90	-.289E+02	-.270E+02	-.235E+02	-.215E+02	-.213E+02
.94	-.933E+01	-.713E+01	-.381E+01	-.255E+01	-.258E+01
.98	.941E+02	.904E+02	.836E+02	.789E+02	.782E+02
1.00	.215E+03	.201E+03	.179E+03	.167E+03	.165E+03

Table 1. Continued

T=60°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.599E+03	.509E+03	.370E+03	.307E+03	.303E+03
-.98	.392E+03	.345E+03	.268E+03	.226E+03	.223E+03
-.94	.127E+03	.126E+03	.120E+03	.108E+03	.106E+03
-.90	.133E+02	.260E+02	.430E+02	.463E+02	.453E+02
-.80	-.218E+02	-.143E+02	-.702E+00	.703E+01	.734E+01
-.70	-.855E+01	-.602E+01	-.466E+00	.419E+01	.473E+01
-.60	-.317E+01	-.235E+01	-.845E-01	.259E+01	.311E+01
-.40	-.535E+00	-.482E+00	-.134E+00	.673E+00	.989E+00
-.20	-.961E-01	-.108E+00	-.841E-01	.910E-01	.209E+00
0.00	-.413E-01	-.692E-01	-.147E+00	-.250E+00	-.275E+00
.20	-.153E+00	-.256E+00	-.551E+00	-.990E+00	-.114E+01
.40	-.882E+00	-.128E+01	-.225E+01	-.334E+01	-.361E+01
.60	-.517E+01	-.643E+01	-.894E+01	-.109E+02	-.111E+02
.70	-.126E+02	-.144E+02	-.176E+02	-.195E+02	-.195E+02
.80	-.302E+02	-.318E+02	-.341E+02	-.345E+02	-.342E+02
.90	-.672E+02	-.657E+02	-.620E+02	-.583E+02	-.575E+02
.94	-.892E+02	-.848E+02	-.764E+02	-.702E+02	-.692E+02
.98	-.118E+03	-.109E+03	-.934E+02	-.839E+02	-.827E+02
1.00	-.139E+03	-.126E+03	-.104E+03	-.922E+02	-.910E+02
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	-.196E+04	-.191E+04	-.183E+04	-.181E+04	-.181E+04
-.98	-.685E+03	-.691E+03	-.700E+03	-.706E+03	-.707E+03
-.94	.228E+03	.215E+03	.194E+03	.185E+03	.184E+03
-.90	.288E+03	.290E+03	.294E+03	.295E+03	.295E+03
-.80	.525E+02	.554E+02	.609E+02	.648E+02	.653E+02
-.70	.298E+01	.206E+01	.640E+00	.188E+00	.266E+00
-.60	.437E+00	-.698E-01	-.106E+01	-.171E+01	-.174E+01
-.40	.161E+00	.127E+00	.122E-01	-.148E+00	-.180E+00
-.20	.261E-01	.233E-01	.145E-02	-.556E-01	-.803E-01
0.00	-.299E-02	-.810E-02	-.271E-01	-.670E-01	-.863E-01
.20	-.442E-01	-.669E-01	-.125E+00	-.194E+00	-.213E+00
.40	-.263E+00	-.342E+00	-.503E+00	-.626E+00	-.637E+00
.60	-.152E+01	-.167E+01	-.189E+01	-.193E+01	-.189E+01
.70	-.397E+01	-.403E+01	-.401E+01	-.378E+01	-.370E+01
.80	-.126E+02	-.123E+02	-.114E+02	-.105E+02	-.103E+02
.90	-.279E+02	-.255E+02	-.217E+02	-.198E+02	-.197E+02
.94	-.776E+01	-.532E+01	-.199E+01	-.107E+01	-.115E+01
.98	.923E+02	.879E+02	.801E+02	.752E+02	.746E+02
1.00	.206E+03	.191E+03	.167E+03	.156E+03	.154E+03

Table 1. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.581E+03	.481E+03	.337E+03	.278E+03	.275E+03
-.98	.384E+03	.332E+03	.249E+03	.208E+03	.205E+03
-.94	.129E+03	.127E+03	.118E+03	.105E+03	.103E+03
-.90	.164E+02	.302E+02	.468E+02	.481E+02	.469E+02
-.80	-.213E+02	-.126E+02	.226E+01	.961E+01	.969E+01
-.70	-.845E+01	-.542E+01	.103E+01	.590E+01	.630E+01
-.60	-.312E+01	-.209E+01	.725E+00	.375E+01	.421E+01
-.40	-.547E+00	-.459E+00	.501E-01	.110E+01	.144E+01
-.20	-.103E+00	-.114E+00	-.599E-01	.202E+00	.345E+00
0.00	-.480E-01	-.848E-01	-.187E+00	-.314E+00	-.337E+00
.20	-.176E+00	-.311E+00	-.699E+00	-.126E+01	-.142E+01
.40	-.972E+00	-.147E+01	-.266E+01	-.394E+01	-.420E+01
.60	-.548E+01	-.696E+01	-.984E+01	-.118E+02	-.120E+02
.70	-.130E+02	-.151E+02	-.187E+02	-.204E+02	-.203E+02
.80	-.307E+02	-.325E+02	-.346E+02	-.346E+02	-.342E+02
.90	-.668E+02	-.649E+02	-.605E+02	-.563E+02	-.555E+02
.94	-.879E+02	-.829E+02	-.734E+02	-.669E+02	-.659E+02
.98	-.116E+03	-.105E+03	-.884E+02	-.789E+02	-.778E+02
1.00	-.136E+03	-.121E+03	-.975E+02	-.861E+02	-.850E+02
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	-.187E+04	-.180E+04	-.172E+04	-.170E+04	-.170E+04
-.98	-.679E+03	-.683E+03	-.691E+03	-.695E+03	-.696E+03
-.94	.206E+03	.190E+03	.166E+03	.156E+03	.156E+03
-.90	.282E+03	.283E+03	.285E+03	.285E+03	.285E+03
-.80	.561E+02	.599E+02	.669E+02	.713E+02	.717E+02
-.70	.308E+01	.214E+01	.859E+00	.692E+00	.806E+00
-.60	.275E+00	-.351E+00	-.152E+01	-.218E+01	-.219E+01
-.40	.160E+00	.115E+00	-.335E-01	-.219E+00	-.246E+00
-.20	.267E-01	.218E-01	-.114E-01	-.880E-01	-.115E+00
0.00	-.385E-02	-.110E-01	-.378E-01	-.922E-01	-.115E+00
.20	-.492E-01	-.779E-01	-.150E+00	-.233E+00	-.252E+00
.40	-.282E+00	-.375E+00	-.560E+00	-.684E+00	-.689E+00
.60	-.156E+01	-.172E+01	-.194E+01	-.193E+01	-.188E+01
.70	-.404E+01	-.410E+01	-.403E+01	-.373E+01	-.364E+01
.80	-.128E+02	-.124E+02	-.113E+02	-.104E+02	-.102E+02
.90	-.266E+02	-.239E+02	-.198E+02	-.179E+02	-.178E+02
.94	-.554E+01	-.287E+01	.422E+00	.103E+01	.926E+00
.98	.903E+02	.852E+02	.765E+02	.716E+02	.710E+02
1.00	.196E+03	.179E+03	.154E+03	.143E+03	.142E+03

Table 1. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(N_0/\beta)$		
-1.0	-.476E+02	-.425E+02	-.338E+02	-.287E+02	-.283E+02
-.98	-.315E+02	-.289E+02	-.242E+02	-.211E+02	-.207E+02
-.94	-.113E+02	-.113E+02	-.110E+02	-.103E+02	-.102E+02
-.90	-.270E+01	-.337E+01	-.442E+01	-.478E+01	-.473E+01
-.80	.629E+00	.225E+00	-.555E+00	-.111E+01	-.116E+01
-.70	.221E+00	.697E-01	-.266E+00	-.582E+00	-.639E+00
-.60	.683E-01	.106E-01	-.135E+00	-.310E+00	-.356E+00
-.40	.102E-01	.208E-02	-.250E-01	-.746E-01	-.972E-01
-.20	.156E-02	.427E-03	-.460E-02	-.176E-01	-.267E-01
0.00	.675E-04	-.204E-03	-.149E-02	-.544E-02	-.906E-02
.20	-.112E-02	-.174E-02	-.361E-02	-.710E-02	-.940E-02
.40	-.755E-02	-.102E-01	-.169E-01	-.253E-01	-.283E-01
.60	-.491E-01	-.588E-01	-.797E-01	-.989E-01	-.103E+00
.70	-.125E+00	-.141E+00	-.172E+00	-.195E+00	-.197E+00
.80	-.316E+00	-.334E+00	-.364E+00	-.378E+00	-.378E+00
.90	-.766E+00	-.761E+00	-.743E+00	-.715E+00	-.707E+00
.94	-.108E+01	-.105E+01	-.978E+00	-.915E+00	-.903E+00
.98	-.156E+01	-.147E+01	-.130E+01	-.118E+01	-.116E+01
1.00	-.193E+01	-.178E+01	-.152E+01	-.135E+01	-.133E+01
			$\sigma(x,t)/(N_0/\beta)$		
-1.0	.127E+03	.124E+03	.120E+03	.119E+03	.119E+03
-.98	.415E+02	.419E+02	.428E+02	.433E+02	.434E+02
-.94	-.160E+02	-.154E+02	-.144E+02	-.138E+02	-.138E+02
-.90	-.179E+02	-.182E+02	-.185E+02	-.188E+02	-.188E+02
-.80	-.276E+01	-.288E+01	-.314E+01	-.337E+01	-.341E+01
-.70	-.849E-01	-.335E-01	.567E-01	.103E+00	.102E+00
-.60	.204E-02	.280E-01	.815E-01	.124E+00	.129E+00
-.40	-.335E-02	-.599E-03	.696E-02	.171E-01	.199E-01
-.20	-.518E-03	-.137E-03	.125E-02	.403E-02	.535E-02
0.00	-.136E-03	-.116E-03	.444E-04	.606E-03	.105E-02
.20	-.387E-03	-.530E-03	-.867E-03	-.119E-02	-.117E-02
.40	-.243E-02	-.302E-02	-.432E-02	-.551E-02	-.568E-02
.60	-.158E-01	-.172E-01	-.199E-01	-.213E-01	-.212E-01
.70	-.422E-01	-.434E-01	-.447E-01	-.440E-01	-.433E-01
.80	-.122E+00	-.119E+00	-.111E+00	-.102E+00	-.100E+00
.90	-.217E+00	-.197E+00	-.159E+00	-.135E+00	-.132E+00
.94	-.182E-01	.930E-03	.309E-01	.435E-01	.433E-01
.98	.796E+00	.758E+00	.686E+00	.632E+00	.624E+00
1.00	.166E+01	.154E+01	.133E+01	.120E+01	.118E+01

Table 2. Adhesive stresses for a single lap joint subjected to axial loading ($N_0 \neq 0$, $Q_0=M_0=\Delta T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ= 12.7mm, and $\beta=2.54\times10^{-2}$ m.

T=43°C

$\tau(x,t)/(N_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.460E+02	-.404E+02	-.312E+02	-.263E+02	-.259E+02
-.98	-.307E+02	-.278E+02	-.227E+02	-.196E+02	-.193E+02
-.94	-.113E+02	-.113E+02	-.108E+02	-.100E+02	-.983E+01
-.90	-.293E+01	-.365E+01	-.467E+01	-.493E+01	-.486E+01
-.80	.518E+00	.582E-01	-.800E+00	-.135E+01	-.138E+01
-.70	.179E+00	-.169E-02	-.393E+00	-.735E+00	-.786E+00
-.60	.515E-01	-.208E-01	-.201E+00	-.405E+00	-.451E+00
-.40	.792E-02	-.342E-02	-.409E-01	-.106E+00	-.133E+00
-.20	.126E-02	-.529E-03	-.828E-02	-.275E-01	-.396E-01
0.00	-.546E-05	-.449E-03	-.254E-02	-.881E-02	-.141E-01
.20	-.132E-02	-.216E-02	-.473E-02	-.969E-02	-.129E-01
.40	-.843E-02	-.118E-01	-.201E-01	-.304E-01	-.339E-01
.60	-.524E-01	-.639E-01	-.881E-01	-.109E+00	-.113E+00
.70	-.131E+00	-.149E+00	-.183E+00	-.206E+00	-.208E+00
.80	-.323E+00	-.342E+00	-.373E+00	-.385E+00	-.383E+00
.90	-.764E+00	-.757E+00	-.732E+00	-.699E+00	-.691E+00
.94	-.107E+01	-.103E+01	-.950E+00	-.881E+00	-.869E+00
.98	-.153E+01	-.143E+01	-.124E+01	-.112E+01	-.110E+01
1.00	-.187E+01	-.171E+01	-.143E+01	-.127E+01	-.125E+01

$\sigma(x,t)/(N_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.124E+03	.121E+03	.117E+03	.116E+03	.116E+03
-.98	.415E+02	.420E+02	.428E+02	.434E+02	.435E+02
-.94	-.154E+02	-.147E+02	-.136E+02	-.131E+02	-.130E+02
-.90	-.179E+02	-.181E+02	-.185E+02	-.187E+02	-.187E+02
-.80	-.285E+01	-.301E+01	-.332E+01	-.358E+01	-.362E+01
-.70	-.719E-01	-.159E-01	.765E-01	.114E+00	.110E+00
-.60	.115E-01	.422E-01	.103E+00	.148E+00	.151E+00
-.40	-.256E-02	.996E-03	.106E-01	.226E-01	.253E-01
-.20	-.414E-03	.138E-03	.212E-02	.584E-02	.738E-02
0.00	-.134E-03	-.900E-04	.188E-03	.105E-02	.166E-02
.20	-.435E-03	-.611E-03	-.101E-02	-.133E-02	-.123E-02
.40	-.263E-02	-.334E-02	-.484E-02	-.611E-02	-.621E-02
.60	-.163E-01	-.179E-01	-.207E-01	-.219E-01	-.216E-01
.70	-.427E-01	-.439E-01	-.449E-01	-.435E-01	-.427E-01
.80	-.121E+00	-.118E+00	-.108E+00	-.982E-01	-.961E-01
.90	-.208E+00	-.185E+00	-.145E+00	-.121E+00	-.119E+00
.94	-.769E-02	.126E-01	.425E-01	.527E-01	.521E-01
.98	.781E+00	.738E+00	.658E+00	.602E+00	.594E+00
1.00	.160E+01	.147E+01	.125E+01	.112E+01	.111E+01

Table 2. Continued

T=60°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(N_0\beta)$		
-1.0	-.439E+02	-.376E+02	-.278E+02	-.232E+02	-.229E+02
-.98	-.297E+02	-.263E+02	-.207E+02	-.177E+02	-.174E+02
-.94	-.114E+02	-.112E+02	-.105E+02	-.952E+01	-.934E+01
-.90	-.321E+01	-.398E+01	-.497E+01	-.505E+01	-.495E+01
-.80	.374E+00	-.176E+00	-.114E+01	-.166E+01	-.167E+01
-.70	.121E+00	-.107E+00	-.583E+00	-.951E+00	-.988E+00
-.60	.272E-01	-.700E-01	-.306E+00	-.548E+00	-.591E+00
-.40	.421E-02	-.132E-01	-.699E-01	-.162E+00	-.194E+00
-.20	.706E-03	-.244E-02	-.158E-01	-.471E-01	-.641E-01
0.00	-.147E-03	-.961E-03	-.483E-02	-.161E-01	-.247E-01
.20	-.163E-02	-.287E-02	-.685E-02	-.148E-01	-.199E-01
.40	-.970E-02	-.142E-01	-.254E-01	-.388E-01	-.432E-01
.60	-.569E-01	-.713E-01	-.101E+00	-.124E+00	-.127E+00
.70	-.138E+00	-.159E+00	-.198E+00	-.221E+00	-.222E+00
.80	-.331E+00	-.352E+00	-.383E+00	-.391E+00	-.388E+00
.90	-.761E+00	-.750E+00	-.715E+00	-.675E+00	-.665E+00
.94	-.105E+01	-.101E+01	-.908E+00	-.833E+00	-.822E+00
.98	-.149E+01	-.137E+01	-.116E+01	-.103E+01	-.102E+01
1.00	-.181E+01	-.162E+01	-.132E+01	-.116E+01	-.115E+01
			$\sigma(x,t)/(N_0/\beta)$		
-1.0	.121E+03	.118E+03	.114E+03	.113E+03	.113E+03
-.98	.415E+02	.420E+02	.429E+02	.435E+02	.436E+02
-.94	-.146E+02	-.138E+02	-.126E+02	-.120E+02	-.120E+02
-.90	-.178E+02	-.181E+02	-.184E+02	-.186E+02	-.186E+02
-.80	-.299E+01	-.319E+01	-.358E+01	-.386E+01	-.390E+01
-.70	-.566E-01	.577E-02	.978E-01	.119E+00	.111E+00
-.60	.244E-01	.627E-01	.135E+00	.179E+00	.179E+00
-.40	-.138E-02	.358E-02	.166E-01	.310E-01	.333E-01
-.20	-.237E-03	.643E-03	.372E-02	.903E-02	.108E-01
0.00	-.123E-03	-.275E-04	.504E-03	.198E-02	.282E-02
.20	-.502E-03	-.733E-03	-.122E-02	-.145E-02	-.119E-02
.40	-.290E-02	-.379E-02	-.562E-02	-.690E-02	-.684E-02
.60	-.169E-01	-.188E-01	-.217E-01	-.223E-01	-.219E-01
.70	-.434E-01	-.445E-01	-.449E-01	-.424E-01	-.414E-01
.80	-.121E+00	-.116E+00	-.103E+00	-.919E-01	-.900E-01
.90	-.197E+00	-.170E+00	-.126E+00	-.103E+00	-.102E+00
.94	.501E-02	.270E-01	.561E-01	.627E-01	.616E-01
.98	.761E+00	.709E+00	.618E+00	.561E+00	.553E+00
1.00	.153E+01	.138E+01	.114E+01	.102E+01	.101E+01

Table 2. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(N_0\beta)$		
-1.0	-.426E+02	-.356E+02	-.254E+02	-.211E+02	-.209E+02
-.98	-.291E+02	-.253E+02	-.193E+02	-.163E+02	-.161E+02
-.94	-.115E+02	-.112E+02	-.102E+02	-.916E+01	-.900E+01
-.90	-.339E+01	-.423E+01	-.515E+01	-.509E+01	-.499E+01
-.80	.324E+00	-.305E+00	-.135E+01	-.183E+01	-.182E+01
-.70	.974E-01	-.173E+00	-.716E+00	-.109E+01	-.112E+01
-.60	.133E-01	-.106E+00	-.390E+00	-.658E+00	-.695E+00
-.40	.180E-02	-.213E-01	-.967E-01	-.212E+00	-.246E+00
-.20	.313E-03	-.421E-02	-.236E-01	-.672E-01	-.881E-01
0.00	-.261E-03	-.148E-02	-.743E-02	-.245E-01	-.361E-01
.20	-.188E-02	-.355E-02	-.908E-02	-.204E-01	-.274E-01
.40	-.107E-01	-.164E-01	-.303E-01	-.466E-01	-.518E-01
.60	-.604E-01	-.775E-01	-.111E+00	-.135E+00	-.138E+00
.70	-.143E+00	-.168E+00	-.210E+00	-.232E+00	-.232E+00
.80	-.337E+00	-.360E+00	-.391E+00	-.393E+00	-.389E+00
.90	-.758E+00	-.743E+00	-.699E+00	-.653E+00	-.644E+00
.94	-.104E+01	-.984E+00	-.874E+00	-.795E+00	-.783E+00
.98	-.146E+01	-.132E+01	-.109E+01	-.972E+00	-.958E+00
1.00	-.176E+01	-.155E+01	-.123E+01	-.108E+01	-.107E+01
			$\sigma(x,t)/(N_0/\beta)$		
-1.0	.115E+03	.111E+03	.107E+03	.106E+03	.106E+03
-.98	.412E+02	.416E+02	.424E+02	.429E+02	.430E+02
-.94	-.133E+02	-.123E+02	-.108E+02	-.102E+02	-.102E+02
-.90	-.175E+02	-.177E+02	-.179E+02	-.180E+02	-.180E+02
-.80	-.321E+01	-.348E+01	-.397E+01	-.429E+01	-.432E+01
-.70	-.603E-01	.326E-02	.841E-01	.844E-01	.740E-01
-.60	.363E-01	.828E-01	.166E+00	.209E+00	.208E+00
-.40	-.677E-03	.553E-02	.215E-01	.375E-01	.391E-01
-.20	-.122E-03	.107E-02	.523E-02	.119E-01	.136E-01
0.00	-.117E-03	.321E-04	.841E-03	.294E-02	.395E-02
.20	-.556E-03	-.838E-03	-.139E-02	-.145E-02	-.103E-02
.40	-.311E-02	-.417E-02	-.625E-02	-.743E-02	-.720E-02
.60	-.174E-01	-.195E-01	-.224E-01	-.224E-01	-.218E-01
.70	-.442E-01	-.453E-01	-.450E-01	-.416E-01	-.405E-01
.80	-.121E+00	-.115E+00	-.100E+00	-.879E-01	-.861E-01
.90	-.185E+00	-.154E+00	-.107E+00	-.865E-01	-.853E-01
.94	.212E-01	.445E-01	.718E-01	.748E-01	.734E-01
.98	.741E+00	.682E+00	.582E+00	.524E+00	.517E+00
1.00	.145E+01	.128E+01	.104E+01	.921E+00	.910E+00

Table 2. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.639E+03	-.566E+03	-.443E+03	-.372E+03	-.366E+03
-.98	-.405E+03	-.369E+03	-.304E+03	-.262E+03	-.257E+03
-.94	-.114E+03	-.116E+03	-.114E+03	-.107E+03	-.105E+03
-.90	.337E+01	-.730E+01	-.243E+02	-.312E+02	-.308E+02
-.80	.333E+02	.279E+02	.173E+02	.945E+01	.859E+01
-.70	.181E+02	.165E+02	.128E+02	.897E+01	.822E+01
-.60	.123E+02	.118E+02	.105E+02	.873E+01	.820E+01
-.40	.945E+01	.945E+01	.934E+01	.896E+01	.874E+01
-.20	.903E+01	.904E+01	.904E+01	.897E+01	.889E+01
0.00	.896E+01	.896E+01	.897E+01	.896E+01	.893E+01
.20	.895E+01	.895E+01	.895E+01	.895E+01	.895E+01
.40	.895E+01	.895E+01	.895E+01	.896E+01	.895E+01
.60	.897E+01	.897E+01	.898E+01	.898E+01	.898E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.900E+01
.80	.910E+01	.909E+01	.907E+01	.905E+01	.904E+01
.90	.903E+01	.901E+01	.897E+01	.895E+01	.895E+01
.94	.875E+01	.875E+01	.876E+01	.877E+01	.878E+01
.98	.831E+01	.836E+01	.846E+01	.852E+01	.853E+01
1.00	.819E+01	.826E+01	.838E+01	.846E+01	.847E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	.199E+04	.195E+04	.188E+04	.185E+04	.185E+04
-.98	.649E+03	.654E+03	.664E+03	.671E+03	.672E+03
-.94	-.257E+03	-.248E+03	-.232E+03	-.222E+03	-.221E+03
-.90	-.289E+03	-.292E+03	-.296E+03	-.299E+03	-.299E+03
-.80	-.480E+02	-.497E+02	-.533E+02	-.564E+02	-.569E+02
-.70	-.321E+01	-.247E+01	-.113E+01	-.364E+00	-.357E+00
-.60	-.695E+00	-.363E+00	.339E+00	.933E+00	.101E+01
-.40	-.164E+00	-.149E+00	-.923E-01	.642E-02	.393E-01
-.20	-.251E-01	-.261E-01	-.220E-01	-.155E-02	.118E-01
0.00	-.384E-02	-.453E-02	-.497E-02	-.124E-02	.325E-02
.20	-.415E-03	-.566E-03	-.771E-03	-.124E-03	.124E-02
.40	.108E-02	.117E-02	.130E-02	.141E-02	.174E-02
.60	.697E-02	.658E-02	.548E-02	.418E-02	.400E-02
.70	.301E-01	.301E-01	.300E-01	.297E-01	.298E-01
.80	.203E+00	.213E+00	.231E+00	.243E+00	.245E+00
.90	.259E+00	.240E+00	.205E+00	.185E+00	.184E+00
.94	-.136E+01	-.143E+01	-.154E+01	-.160E+01	-.161E+01
.98	-.753E+01	-.751E+01	-.747E+01	-.744E+01	-.744E+01
1.00	-.142E+02	-.139E+02	-.135E+02	-.132E+02	-.132E+02

Table 3. Adhesive stresses for a single lap joint subjected to transverse shear loading ($Q_0 \neq 0$, $N_0=M_0=\Delta T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=2.54\times10^{-2}$m.

T=43°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.615E+03	-.536E+03	-.407E+03	-.339E+03	-.333E+03
-.98	-.394E+03	-.354E+03	-.284E+03	-.242E+03	-.237E+03
-.94	-.115E+03	-.116E+03	-.113E+03	-.104E+03	-.102E+03
-.90	-.244E+00	-.117E+02	-.288E+02	-.342E+02	-.335E+02
-.80	.319E+02	.257E+02	.139E+02	.601E+01	.537E+01
-.70	.177E+02	.158E+02	.113E+02	.713E+01	.645E+01
-.60	.122E+02	.116E+02	.992E+01	.775E+01	.720E+01
-.40	.946E+01	.944E+01	.974E+01	.870E+01	.843E+01
-.20	.903E+01	.904E+01	.903E+01	.890E+01	.880E+01
0.00	.896E+01	.896E+01	.897E+01	.894E+01	.890E+01
.20	.895E+01	.895E+01	.895E+01	.895E+01	.894E+01
.40	.895E+01	.895E+01	.896E+01	.896E+01	.895E+01
.60	.897E+01	.897E+01	.898E+01	.898E+01	.897E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.900E+01
.80	.909E+01	.908E+01	.906E+01	.904E+01	.904E+01
.90	.902E+01	.900E+01	.896E+01	.894E+01	.895E+01
.94	.875E+01	.875E+01	.876E+01	.877E+01	.878E+01
.98	.832E+01	.838E+01	.848E+01	.855E+01	.856E+01
1.00	.821E+01	.828E+01	.841E+01	.850E+01	.851E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	.195E+04	.190E+04	.183E+04	.180E+04	.180E+04
-.98	.649E+03	.654E+03	.664E+03	.670E+03	.671E+03
-.94	-.248E+03	-.237E+03	-.219E+03	-.210E+03	-.209E+03
-.90	-.289E+03	-.291E+03	-.295E+03	-.297E+03	-.298E+03
-.80	-.494E+02	-.516E+02	-.559E+02	-.593E+02	-.599E+02
-.70	-.303E+01	-.221E+01	-.820E+00	-.163E+00	-.195E+00
-.60	-.572E+00	-.175E+00	.641E+00	.127E+01	.133E+01
-.40	-.161E+00	-.139E+00	-.608E-01	.619E-01	.953E-01
-.20	-.258E-01	-.259E-01	-.172E-01	.132E-01	.295E-01
0.00	-.412E-02	-.478E-02	-.443E-02	.238E-02	.874E-02
.20	-.470E-03	-.633E-03	-.725E-03	.733E-03	.294E-02
.40	.113E-02	.123E-02	.136E-02	.160E-02	.226E-02
.60	.695E-02	.643E-02	.503E-02	.357E-02	.353E-02
.70	.311E-01	.313E-01	.313E-01	.313E-01	.316E-01
.80	.211E+00	.222E+00	.243E+00	.256E+00	.257E+00
.90	.239E+00	.216E+00	.177E+00	.156E+00	.154E+00
.94	-.141E+01	-.149E+01	-.161E+01	-.167E+01	-.167E+01
.98	-.751E+01	-.748E+01	-.743E+01	-.741E+01	-.740E+01
1.00	-.140E+02	-.137E+02	-.132E+02	-.130E+02	-.130E+02

Table 3. Continued

T=60°C

$\tau(x,t)/(Q_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.586E+03	-.497E+03	-.359E+03	-.296E+03	-.292E+03
-.98	-.380E+03	-.333E+03	-.257E+03	-.215E+03	-.212E+03
-.94	-.117E+03	-.116E+03	-.110E+03	-.987E+02	-.965E+02
-.90	-.468E+01	-.173E+02	-.341E+02	-.372E+02	-.363E+02
-.80	.300E+02	.225E+02	.913E+01	.151E+01	.120E+01
-.70	.172E+02	.146E+02	.911E+01	.450E+01	.396E+01
-.60	.120E+02	.112E+02	.888E+01	.621E+01	.569E+01
-.40	.946E+01	.940E+01	.904E+01	.821E+01	.787E+01
-.20	.904E+01	.904E+01	.899E+01	.876E+01	.860E+01
0.00	.896E+01	.897E+01	.896E+01	.890E+01	.884E+01
.20	.895E+01	.895E+01	.895E+01	.894E+01	.891E+01
.40	.895E+01	.895E+01	.896E+01	.895E+01	.895E+01
.60	.897E+01	.898E+01	.898E+01	.898E+01	.897E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.900E+01
.80	.909E+01	.908E+01	.905E+01	.903E+01	.903E+01
.90	.901E+01	.898E+01	.894E+01	.893E+01	.893E+01
.94	.874E+01	.874E+01	.876E+01	.878E+01	.879E+01
.98	.833E+01	.840E+01	.852E+01	.859E+01	.860E+01
1.00	.823E+01	.831E+01	.846E+01	.855E+01	.855E+01

$\sigma(x,t)/(Q_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.189E+04	.184E+04	.177E+04	.174E+04	.175E+04
-.98	.648E+03	.654E+03	.664E+03	.670E+03	.671E+03
-.94	-.236E+03	-.223E+03	-.203E+03	-.194E+03	-.193E+03
-.90	-.287E+03	-.290E+03	-.293E+03	-.295E+03	-.295E+03
-.80	-.514E+02	-.542E+02	-.596E+02	-.635E+02	-.639E+02
-.70	-.282E+01	-.190E+01	-.474E+00	-.195E-01	-.976E-01
-.60	-.403E+00	.101E+00	.108E+01	.172E+01	.175E+01
-.40	-.155E+00	-.120E+00	-.532E-02	.151E+00	.181E+00
-.20	-.263E-01	-.244E-01	-.634E-02	.417E-01	.611E-01
0.00	-.445E-02	-.492E-02	-.257E-02	.107E-01	.200E-01
.20	-.541E-03	-.700E-03	-.410E-03	.307E-02	.692E-02
.40	.120E-02	.130E-02	.146E-02	.219E-02	.358E-02
.60	.688E-02	.614E-02	.429E-02	.275E-02	.307E-02
.70	.325E-01	.329E-01	.334E-01	.339E-01	.344E-01
.80	.221E+00	.235E+00	.259E+00	.272E+00	.273E+00
.90	.213E+00	.184E+00	.137E+00	.115E+00	.115E+00
.94	-.148E+01	-.157E+01	-.170E+01	-.175E+01	-.175E+01
.98	-.748E+01	-.744E+01	-.738E+01	-.735E+01	-.735E+01
1.00	-.137E+02	-.134E+02	-.128E+02	-.126E+02	-.126E+02

Table 3. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.568E+03	-.469E+03	-.326E+03	-.267E+03	-.264E+03
-.98	-.372E+03	-.320E+03	-.238E+03	-.198E+03	-.195E+03
-.94	-.119E+03	-.117E+03	-.108E+03	-.950E+02	-.930E+02
-.90	-.770E+01	-.214E+02	-.377E+02	-.390E+02	-.379E+02
-.80	.295E+02	.209E+02	.619E+01	-.105E+01	-.112E+01
-.70	.170E+02	.140E+02	.760E+01	.278E+01	.238E+01
-.60	.119E+02	.109E+02	.806E+01	.504E+01	.457E+01
-.40	.947E+01	.937E+01	.884E+01	.776E+01	.739E+01
-.20	.904E+01	.904E+01	.895E+01	.860E+01	.840E+01
0.00	.896E+01	.897E+01	.896E+01	.885E+01	.876E+01
.20	.895E+01	.895E+01	.895E+01	.893E+01	.888E+01
.40	.895E+01	.896E+01	.896E+01	.895E+01	.893E+01
.60	.898E+01	.898E+01	.898E+01	.898E+01	.897E+01
.70	.902E+01	.902E+01	.901E+01	.900E+01	.899E+01
.80	.910E+01	.908E+01	.905E+01	.902E+01	.902E+01
.90	.900E+01	.897E+01	.893E+01	.892E+01	.892E+01
.94	.873E+01	.873E+01	.875E+01	.878E+01	.878E+01
.98	.833E+01	.841E+01	.854E+01	.861E+01	.861E+01
1.00	.823E+01	.832E+01	.849E+01	.857E+01	.858E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	.180E+04	.174E+04	.166E+04	.164E+04	.164E+04
-.98	.642E+03	.647E+03	.654E+03	.659E+03	.660E+03
-.94	-.214E+03	-.198E+03	-.175E+03	-.166E+03	-.165E+03
-.90	-.282E+03	-.283E+03	-.284E+03	-.285E+03	-.285E+03
-.80	-.549E+02	-.586E+02	-.655E+02	-.698E+02	-.702E+02
-.70	-.290E+01	-.195E+01	-.663E+00	-.490E+00	-.603E+00
-.60	-.239E+00	.382E+00	.154E+01	.219E+01	.219E+01
-.40	-.153E+00	-.107E+00	.402E-01	.219E+00	.244E+00
-.20	-.270E-01	-.234E-01	.416E-02	.679E-01	.882E-01
0.00	-.476E-02	-.502E-02	-.361E-03	.198E-01	.313E-01
.20	-.600E-03	-.744E-03	.589E-04	.601E-02	.114E-01
.40	.128E-02	.140E-02	.163E-02	.306E-02	.531E-02
.60	.706E-02	.610E-02	.381E-02	.235E-02	.312E-02
.70	.362E-01	.374E-01	.394E-01	.410E-01	.418E-01
.80	.237E+00	.255E+00	.283E+00	.297E+00	.299E+00
.90	.162E+00	.122E+00	.608E-01	.362E-01	.356E-01
.94	-.159E+01	-.169E+01	-.183E+01	-.188E+01	-.188E+01
.98	-.741E+01	-.736E+01	-.728E+01	-.725E+01	-.725E+01
1.00	-.133E+02	-.128E+02	-.123E+02	-.121E+02	-.121E+02

Table 3. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.148E+03	.137E+03	.118E+03	.106E+03	.105E+03
-.98	.124E+03	.117E+03	.104E+03	.950E+02	.937E+02
-.94	.917E+02	.887E+02	.824E+02	.771E+02	.761E+02
-.90	.679E+02	.671E+02	.648E+02	.621E+02	.614E+02
-.80	.290E+02	.304E+02	.327E+02	.337E+02	.336E+02
-.70	.115E+02	.128E+02	.154E+02	.173E+02	.175E+02
-.60	.449E+01	.534E+01	.714E+01	.878E+01	.910E+01
-.40	.690E+00	.926E+00	.151E+01	.222E+01	.245E+01
-.20	.104E+00	.155E+00	.300E+00	.522E+00	.618E+00
0.00	.432E-10	-.606E-10	-.201E-10	-.859E-11	-.799E-11
.20	-.104E+00	-.155E+00	-.300E+00	-.522E+00	-.618E+00
.40	-.690E+00	-.926E+00	-.151E+01	-.222E+01	-.245E+01
.60	-.449E+01	-.534E+01	-.714E+01	-.878E+01	-.910E+01
.70	-.115E+02	-.128E+02	-.154E+02	-.173E+02	-.175E+02
.80	-.290E+02	-.304E+02	-.327E+02	-.337E+02	-.336E+02
.90	-.679E+02	-.671E+02	-.648E+02	-.621E+02	-.614E+02
.94	-.917E+02	-.887E+02	-.824E+02	-.771E+02	-.761E+02
.98	-.124E+03	-.117E+03	-.104E+03	-.950E+02	-.937E+02
1.00	-.148E+03	-.137E+03	-.118E+03	-.106E+03	-.105E+03
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	.221E+03	.209E+03	.188E+03	.175E+03	.174E+03
-.98	.955E+02	.923E+02	.861E+02	.816E+02	.809E+02
-.94	-.106E+02	-.857E+01	-.529E+01	-.383E+01	-.382E+01
-.90	-.298E+02	-.280E+02	-.248E+02	-.228E+02	-.226E+02
-.80	-.126E+02	-.124E+02	-.118E+02	-.112E+02	-.110E+02
-.70	-.387E+01	-.395E+01	-.402E+01	-.393E+01	-.387E+01
-.60	-.143E+01	-.155E+01	-.176E+01	-.186E+01	-.185E+01
-.40	-.222E+00	-.274E+00	-.389E+00	-.496E+00	-.515E+00
-.20	-.350E-01	-.490E-01	-.849E-01	-.133E+00	-.149E+00
0.00	-.105E-01	-.164E-01	-.335E-01	-.618E-01	-.752E-01
.20	-.350E-01	-.490E-01	-.849E-01	-.133E+00	-.149E+00
.40	-.222E+00	-.274E+00	-.389E+00	-.496E+00	-.515E+00
.60	-.143E+01	-.155E+01	-.176E+01	-.186E+01	-.185E+01
.70	-.387E+01	-.395E+01	-.402E+01	-.393E+01	-.387E+01
.80	-.126E+02	-.124E+02	-.118E+02	-.112E+02	-.110E+02
.90	-.298E+02	-.280E+02	-.248E+02	-.228E+02	-.226E+02
.94	-.106E+02	-.857E+01	-.529E+01	-.383E+01	-.382E+01
.98	.955E+02	.923E+02	.861E+02	.816E+02	.809E+02
1.00	.221E+03	.209E+03	.188E+03	.175E+03	.174E+03

Table 4. Adhesive stresses for a cover plate subjected to bending ($M_0 \neq 0$, $Q_0=N_0=\Delta T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=2.54\times10^{-2}$m.

T=43°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.144E+03	.132E+03	.112E+03	.100E+03	.989E+02
-.98	.122E+03	.114E+03	.996E+02	.903E+02	.890E+02
-.94	.906E+02	.871E+02	.800E+02	.743E+02	.732E+02
-.90	.676E+02	.665E+02	.637E+02	.606E+02	.599E+02
-.80	.295E+02	.310E+02	.333E+02	.341E+02	.339E+02
-.70	.120E+02	.135E+02	.163E+02	.182E+02	.184E+02
-.60	.478E+01	.578E+01	.787E+01	.965E+01	.994E+01
-.40	.769E+00	.106E+01	.179E+01	.265E+01	.289E+01
-.20	.121E+00	.188E+00	.378E+00	.667E+00	.781E+00
0.00	-.161E-11	-.178E-11	-.103E-11	-.161E-12	-.322E-11
.20	-.121E+00	-.188E+00	-.378E+00	-.667E+00	-.781E+00
.40	-.769E+00	-.106E+01	-.179E+01	-.265E+01	-.289E+01
.60	-.478E+01	-.578E+01	-.787E+01	-.965E+01	-.994E+01
.70	-.120E+02	-.135E+02	-.163E+02	-.182E+02	-.184E+02
.80	-.295E+02	-.310E+02	-.333E+02	-.341E+02	-.339E+02
.90	-.676E+02	-.665E+02	-.637E+02	-.606E+02	-.599E+02
.94	-.906E+02	-.871E+02	-.800E+02	-.743E+02	-.732E+02
.98	-.122E+03	-.114E+03	-.996E+02	-.903E+02	-.890E+02
1.00	-.144E+03	-.132E+03	-.112E+03	-.100E+03	-.989E+02
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	.215E+03	.201E+03	.179E+03	.167E+03	.165E+03
-.98	.941E+02	.904E+02	.836E+02	.789E+02	.782E+02
-.94	-.933E+01	-.713E+01	-.381E+01	-.255E+01	-.258E+01
-.90	-.289E+02	-.270E+02	-.235E+02	-.215E+02	-.213E+02
-.80	-.126E+02	-.123E+02	-.117E+02	-.109E+02	-.107E+02
-.70	-.391E+01	-.399E+01	-.403E+01	-.388E+01	-.381E+01
-.60	-.147E+01	-.160E+01	-.182E+01	-.190E+01	-.188E+01
-.40	-.240E+00	-.302E+00	-.435E+00	-.552E+00	-.569E+00
-.20	-.396E-01	-.574E-01	-.103E+00	-.161E+00	-.180E+00
0.00	-.124E-01	-.203E-01	-.432E-01	-.808E-01	-.971E-01
.20	-.396E-01	-.574E-01	-.103E+00	-.161E+00	-.180E+00
.40	-.240E+00	-.302E+00	-.435E+00	-.552E+00	-.569E+00
.60	-.147E+01	-.160E+01	-.182E+01	-.190E+01	-.188E+01
.70	-.391E+01	-.399E+01	-.403E+01	-.388E+01	-.381E+01
.80	-.126E+02	-.123E+02	-.117E+02	-.109E+02	-.107E+02
.90	-.289E+02	-.270E+02	-.235E+02	-.215E+02	-.213E+02
.94	-.933E+01	-.713E+01	-.381E+01	-.255E+01	-.258E+01
.98	.941E+02	.904E+02	.836E+02	.789E+02	.782E+02
1.00	.215E+03	.201E+03	.179E+03	.167E+03	.165E+03

Table 4. Continued

T=60°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.139E+03	.126E+03	.104E+03	.922E+02	.910E+02
-.98	.118E+03	.109E+03	.934E+02	.839E+02	.827E+02
-.94	.892E+02	.848E+02	.764E+02	.702E+02	.692E+02
-.90	.672E+02	.657E+02	.620E+02	.583E+02	.575E+02
-.80	.302E+02	.318E+02	.341E+02	.345E+02	.342E+02
-.70	.126E+02	.144E+02	.176E+02	.195E+02	.195E+02
-.60	.517E+01	.642E+01	.894E+01	.109E+02	.111E+02
-.40	.881E+00	.128E+01	.224E+01	.332E+01	.358E+01
-.20	.146E+00	.243E+00	.516E+00	.917E+00	.105E+01
0.00	-.629E-08	.471E-08	-.234E-08	-.976E-09	-.208E-09
.20	-.146E+00	-.243E+00	-.516E+00	-.917E+00	-.105E+01
.40	-.881E+00	-.128E+01	-.224E+01	-.332E+01	-.358E+01
.60	-.517E+01	-.642E+01	-.894E+01	-.109E+02	-.111E+02
.70	-.126E+02	-.144E+02	-.176E+02	-.195E+02	-.195E+02
.80	-.302E+02	-.318E+02	-.341E+02	-.345E+02	-.342E+02
.90	-.672E+02	-.657E+02	-.620E+02	-.583E+02	-.575E+02
.94	-.892E+02	-.848E+02	-.764E+02	-.702E+02	-.692E+02
.98	-.118E+03	-.109E+03	-.934E+02	-.839E+02	-.827E+02
1.00	-.139E+03	-.126E+03	-.104E+03	-.922E+02	-.910E+02
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	.206E+03	.191E+03	.167E+03	.156E+03	.154E+03
-.98	.923E+02	.879E+02	.801E+02	.752E+02	.746E+02
-.94	-.776E+01	-.532E+01	-.199E+01	-.107E+01	-.115E+01
-.90	-.279E+02	-.255E+02	-.217E+02	-.198E+02	-.197E+02
-.80	-.126E+02	-.123E+02	-.114E+02	-.105E+02	-.103E+02
-.70	-.397E+01	-.403E+01	-.401E+01	-.378E+01	-.370E+01
-.60	-.152E+01	-.167E+01	-.189E+01	-.193E+01	-.190E+01
-.40	-.264E+00	-.343E+00	-.505E+00	-.630E+00	-.642E+00
-.20	-.463E-01	-.706E-01	-.133E+00	-.209E+00	-.229E+00
0.00	-.154E-01	-.267E-01	-.607E-01	-.114E+00	-.135E+00
.20	-.463E-01	-.706E-01	-.133E+00	-.209E+00	-.229E+00
.40	-.264E+00	-.343E+00	-.505E+00	-.630E+00	-.642E+00
.60	-.152E+01	-.167E+01	-.189E+01	-.193E+01	-.190E+01
.70	-.397E+01	-.403E+01	-.401E+01	-.378E+01	-.370E+01
.80	-.126E+02	-.123E+02	-.114E+02	-.105E+02	-.103E+02
.90	-.279E+02	-.255E+02	-.217E+02	-.198E+02	-.197E+02
.94	-.776E+01	-.532E+01	-.199E+01	-.107E+01	-.115E+01
.98	.923E+02	.879E+02	.801E+02	.752E+02	.746E+02
1.00	.206E+03	.191E+03	.167E+03	.156E+03	.154E+03

Table 4. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(M_0/\beta^2)$		
-1.0	.136E+03	.121E+03	.975E+02	.861E+02	.850E+02
-.98	.116E+03	.105E+03	.884E+02	.789E+02	.778E+02
-.94	.879E+02	.829E+02	.734E+02	.669E+02	.659E+02
-.90	.668E+02	.649E+02	.605E+02	.563E+02	.555E+02
-.80	.307E+02	.325E+02	.346E+02	.346E+02	.342E+02
-.70	.130E+02	.151E+02	.186E+02	.203E+02	.203E+02
-.60	.548E+01	.696E+01	.984E+01	.118E+02	.120E+02
-.40	.971E+00	.147E+01	.265E+01	.391E+01	.416E+01
-.20	.167E+00	.293E+00	.651E+00	.116E+01	.130E+01
0.00	-.169E-08	-.159E-09	-.510E-11	-.125E-12	-.694E-11
.20	-.167E+00	-.293E+00	-.651E+00	-.116E+01	-.130E+01
.40	-.971E+00	-.147E+01	-.265E+01	-.391E+01	-.416E+01
.60	-.548E+01	-.696E+01	-.984E+01	-.118E+02	-.120E+02
.70	-.130E+02	-.151E+02	-.186E+02	-.203E+02	-.203E+02
.80	-.307E+02	-.325E+02	-.346E+02	-.346E+02	-.342E+02
.90	-.668E+02	-.649E+02	-.605E+02	-.563E+02	-.555E+02
.94	-.879E+02	-.829E+02	-.734E+02	-.669E+02	-.659E+02
.98	-.116E+03	-.105E+03	-.884E+02	-.789E+02	-.778E+02
1.00	-.136E+03	-.121E+03	-.975E+02	-.861E+02	-.850E+02

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\sigma(x,t)/(M_0/\beta^2)$		
-1.0	.196E+03	.179E+03	.154E+03	.143E+03	.142E+03
-.98	.903E+02	.852E+02	.765E+02	.716E+02	.710E+02
-.94	-.554E+01	-.287E+01	.422E+00	.103E+01	.925E+00
-.90	-.266E+02	-.239E+02	-.198E+02	-.179E+02	-.178E+02
-.80	-.128E+02	-.124E+02	-.113E+02	-.104E+02	-.102E+02
-.70	-.404E+01	-.410E+01	-.403E+01	-.373E+01	-.365E+01
-.60	-.156E+01	-.172E+01	-.194E+01	-.193E+01	-.189E+01
-.40	-.282E+00	-.376E+00	-.563E+00	-.690E+00	-.695E+00
-.20	-.517E-01	-.824E-01	-.160E+00	-.251E+00	-.271E+00
0.00	-.178E-01	-.329E-01	-.781E-01	-.147E+00	-.169E+00
.20	-.517E-01	-.824E-01	-.160E+00	-.251E+00	-.271E+00
.40	-.282E+00	-.376E+00	-.563E+00	-.690E+00	-.695E+00
.60	-.156E+01	-.172E+01	-.194E+01	-.193E+01	-.189E+01
.70	-.404E+01	-.410E+01	-.403E+01	-.373E+01	-.365E+01
.80	-.128E+02	-.124E+02	-.113E+02	-.104E+02	-.102E+02
.90	-.266E+02	-.239E+02	-.198E+02	-.179E+02	-.178E+02
.94	-.554E+01	-.287E+01	.422E+00	.103E+01	.925E+00
.98	.903E+02	.852E+02	.765E+02	.716E+02	.710E+02
1.00	.196E+03	.179E+03	.154E+03	.143E+03	.142E+03

Table 4. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(N_0/\beta)$		
-1.0	.193E+01	.178E+01	.152E+01	.135E+01	.133E+01
-.98	.156E+01	.147E+01	.130E+01	.118E+01	.116E+01
-.94	.108E+01	.105E+01	.978E+00	.915E+00	.903E+00
-.90	.766E+00	.761E+00	.743E+00	.715E+00	.707E+00
-.80	.316E+00	.334E+00	.364E+00	.378E+00	.378E+00
-.70	.125E+00	.141E+00	.172E+00	.194E+00	.197E+00
-.60	.491E-01	.588E-01	.797E-01	.989E-01	.103E+00
-.40	.755E-02	.102E-01	.168E-01	.250E-01	.276E-01
-.20	.113E-02	.171E-02	.333E-02	.586E-02	.697E-02
0.00	.546E-12	-.799E-12	-.268E-12	-.120E-12	-.105E-12
.20	-.113E-02	-.171E-02	-.333E-02	-.586E-02	-.697E-02
.40	-.755E-02	-.102E-01	-.168E-01	-.250E-01	-.276E-01
.60	-.491E-01	-.588E-01	-.797E-01	-.989E-01	-.103E+00
.70	-.125E+00	-.141E+00	-.172E+00	-.194E+00	-.197E+00
.80	-.316E+00	-.334E+00	-.364E+00	-.378E+00	-.378E+00
.90	-.766E+00	-.761E+00	-.743E+00	-.715E+00	-.707E+00
.94	-.108E+01	-.105E+01	-.978E+00	-.915E+00	-.903E+00
.98	-.156E+01	-.147E+01	-.130E+01	-.118E+01	-.116E+01
1.00	-.193E+01	-.178E+01	-.152E+01	-.135E+01	-.133E+01
			$\sigma(x,t)/(N_0/\beta)$		
-1.0	.166E+01	.154E+01	.133E+01	.120E+01	.118E+01
-.98	.796E+00	.758E+00	.686E+00	.632E+00	.624E+00
-.94	-.182E-01	.930E-03	.309E-01	.435E-01	.433E-01
-.90	-.217E+00	-.197E+00	-.159E+00	-.135E+00	-.132E+00
-.80	-.122E+00	-.119E+00	-.111E+00	-.102E+00	-.100E+00
-.70	-.422E-01	-.434E-01	-.447E-01	-.440E-01	-.433E-01
-.60	-.158E-01	-.172E-01	-.199E-01	-.213E-01	-.212E-01
-.40	-.243E-02	-.302E-02	-.433E-02	-.558E-02	-.581E-02
-.20	-.383E-03	-.540E-03	-.945E-03	-.149E-02	-.168E-02
0.00	-.115E-03	-.181E-03	-.372E-03	-.694E-03	-.848E-03
.20	-.383E-03	-.540E-03	-.945E-03	-.149E-02	-.168E-02
.40	-.243E-02	-.302E-02	-.433E-02	-.558E-02	-.581E-02
.60	-.158E-01	-.172E-01	-.199E-01	-.213E-01	-.212E-01
.70	-.422E-01	-.434E-01	-.447E-01	-.440E-01	-.433E-01
.80	-.122E+00	-.119E+00	-.111E+00	-.102E+00	-.100E+00
.90	-.217E+00	-.197E+00	-.159E+00	-.135E+00	-.132E+00
.94	-.182E-01	.930E-03	.309E-01	.435E-01	.433E-01
.98	.796E+00	.758E+00	.686E+00	.632E+00	.624E+00
1.00	.166E+01	.154E+01	.133E+01	.120E+01	.118E+01

Table 5. Adhesive stresses for a cover plate subjected to axial loading ($N_0 \neq 0$, $Q_0=M_0=T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=2.54\times10^{-2}$m.

T=43°C

$\tau(x,t)/(N_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.187E+01	.171E+01	.143E+01	.127E+01	.125E+01
-.98	.153E+01	.143E+01	.124E+01	.112E+01	.110E+01
-.94	.107E+01	.103E+01	.950E+00	.881E+00	.869E+00
-.90	.764E+00	.757E+00	.732E+00	.699E+00	.691E+00
-.80	.323E+00	.342E+00	.373E+00	.385E+00	.383E+00
-.70	.131E+00	.149E+00	.183E+00	.206E+00	.208E+00
-.60	.524E-01	.639E-01	.881E-01	.109E+00	.112E+00
-.40	.843E-02	.118E-01	.200E-01	.299E-01	.328E-01
-.20	.132E-02	.208E-02	.422E-02	.751E-02	.884E-02
0.00	-.480E-13	-.332E-13	-.261E-13	-.238E-13	-.410E-13
.20	-.132E-02	-.208E-02	-.422E-02	-.751E-02	-.884E-02
.40	-.843E-02	-.118E-01	-.200E-01	-.299E-01	-.328E-01
.60	-.524E-01	-.639E-01	-.881E-01	-.109E+00	-.112E+00
.70	-.131E+00	-.149E+00	-.183E+00	-.206E+00	-.208E+00
.80	-.323E+00	-.342E+00	-.373E+00	-.385E+00	-.383E+00
.90	-.764E+00	-.757E+00	-.732E+00	-.699E+00	-.691E+00
.94	-.107E+01	-.103E+01	-.950E+00	-.881E+00	-.869E+00
.98	-.153E+01	-.143E+01	-.124E+01	-.112E+01	-.110E+01
1.00	-.187E+01	-.171E+01	-.143E+01	-.127E+01	-.125E+01

$\sigma(x,t)/(N_0/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.160E+01	.147E+01	.125E+01	.112E+01	.111E+01
-.98	.781E+00	.738E+00	.658E+00	.602E+00	.594E+00
-.94	-.769E-02	.126E-01	.425E-01	.527E-01	.521E-01
-.90	-.208E+00	-.185E+00	-.145E+00	-.121E+00	-.119E+00
-.80	-.121E+00	-.118E+00	-.108E+00	-.982E-01	-.962E-01
-.70	-.427E-01	-.439E-01	-.450E-01	-.436E-01	-.427E-01
-.60	-.163E-01	-.179E-01	-.207E-01	-.219E-01	-.217E-01
-.40	-.263E-02	-.334E-02	-.487E-02	-.623E-02	-.644E-02
-.20	-.435E-03	-.634E-03	-.115E-02	-.182E-02	-.204E-02
0.00	-.136E-03	-.224E-03	-.482E-03	-.910E-03	-.110E-02
.20	-.435E-03	-.634E-03	-.115E-02	-.182E-02	-.204E-02
.40	-.263E-02	-.334E-02	-.487E-02	-.623E-02	-.644E-02
.60	-.163E-01	-.179E-01	-.207E-01	-.219E-01	-.217E-01
.70	-.427E-01	-.439E-01	-.450E-01	-.436E-01	-.427E-01
.80	-.121E+00	-.118E+00	-.108E+00	-.982E-01	-.962E-01
.90	-.208E+00	-.185E+00	-.145E+00	-.121E+00	-.119E+00
.94	-.769E-02	.126E-01	.425E-01	.527E-01	.521E-01
.98	.781E+00	.738E+00	.658E+00	.602E+00	.594E+00
1.00	.160E+01	.147E+01	.125E+01	.112E+01	.111E+01

Table 5. Continued

T=60°C

$\tau(x,t)/(N_o/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.181E+01	.162E+01	.132E+01	.116E+01	.115E+01
-.98	.149E+01	.137E+01	.116E+01	.103E+01	.102E+01
-.94	.105E+01	.101E+01	.908E+00	.833E+00	.821E+00
-.90	.761E+00	.750E+00	.715E+00	.675E+00	.665E+00
-.80	.331E+00	.352E+00	.383E+00	.390E+00	.387E+00
-.70	.138E+00	.159E+00	.198E+00	.221E+00	.222E+00
-.60	.569E-01	.713E-01	.101E+00	.123E+00	.126E+00
-.40	.970E-02	.142E-01	.251E-01	.376E-01	.407E-01
-.20	.161E-02	.269E-02	.579E-02	.104E-01	.120E-01
0.00	-.820E-10	.610E-10	-.304E-10	-.127E-10	-.270E-11
.20	-.161E-02	-.269E-02	-.579E-02	-.104E-01	-.120E-01
.40	-.970E-02	-.142E-01	-.251E-01	-.376E-01	-.407E-01
.60	-.569E-01	-.713E-01	-.101E+00	-.123E+00	-.126E+00
.70	-.138E+00	-.159E+00	-.198E+00	-.221E+00	-.222E+00
.80	-.331E+00	-.352E+00	-.383E+00	-.390E+00	-.387E+00
.90	-.761E+00	-.750E+00	-.715E+00	-.675E+00	-.665E+00
.94	-.105E+01	-.101E+01	-.908E+00	-.833E+00	-.821E+00
.98	-.149E+01	-.137E+01	-.116E+01	-.103E+01	-.102E+01
1.00	-.181E+01	-.162E+01	-.132E+01	-.116E+01	-.115E+01

$\sigma(x,t)/(N_o/\beta)$

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.153E+01	.138E+01	.114E+01	.102E+01	.101E+01
-.98	.761E+00	.709E+00	.618E+00	.561E+00	.553E+00
-.94	.501E-02	.270E-01	.561E-01	.627E-01	.616E-01
-.90	-.197E+00	-.170E+00	-.126E+00	-.103E+00	-.102E+00
-.80	-.121E+00	-.116E+00	-.103E+00	-.919E-01	-.900E-01
-.70	-.434E-01	-.445E-01	-.449E-01	-.424E-01	-.415E-01
-.60	-.169E-01	-.188E-01	-.217E-01	-.224E-01	-.220E-01
-.40	-.290E-02	-.380E-02	-.567E-02	-.715E-02	-.729E-02
-.20	-.510E-03	-.783E-03	-.149E-02	-.236E-02	-.260E-02
0.00	-.169E-03	-.297E-03	-.680E-03	-.130E-02	-.153E-02
.20	-.510E-03	-.783E-03	-.149E-02	-.236E-02	-.260E-02
.40	-.290E-02	-.380E-02	-.567E-02	-.715E-02	-.729E-02
.60	-.169E-01	-.188E-01	-.217E-01	-.224E-01	-.220E-01
.70	-.434E-01	-.445E-01	-.449E-01	-.424E-01	-.415E-01
.80	-.121E+00	-.116E+00	-.103E+00	-.919E-01	-.900E-01
.90	-.197E+00	-.170E+00	-.126E+00	-.103E+00	-.102E+00
.94	.501E-02	.270E-01	.561E-01	.627E-01	.616E-01
.98	.761E+00	.709E+00	.618E+00	.561E+00	.553E+00
1.00	.153E+01	.138E+01	.114E+01	.102E+01	.101E+01

Table 5. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(N_0/\beta)$		
-1.0	.176E+01	.155E+01	.123E+01	.108E+01	.107E+01
-.98	.146E+01	.132E+01	.109E+01	.971E+00	.958E+00
-.94	.104E+01	.984E+00	.874E+00	.795E+00	.783E+00
-.90	.758E+00	.743E+00	.699E+00	.653E+00	.643E+00
-.80	.337E+00	.360E+00	.391E+00	.393E+00	.389E+00
-.70	.143E+00	.168E+00	.210E+00	.231E+00	.231E+00
-.60	.604E-01	.775E-01	.111E+00	.135E+00	.137E+00
-.40	.107E-01	.163E-01	.299E-01	.444E-01	.474E-01
-.20	.184E-02	.326E-02	.732E-02	.131E-01	.148E-01
0.00	-.220E-10	-.208E-11	-.733E-13	.177E-14	-.898E-13
.20	-.184E-02	-.326E-02	-.732E-02	-.131E-01	-.148E-01
.40	-.107E-01	-.163E-01	-.299E-01	-.444E-01	-.474E-01
.60	-.604E-01	-.775E-01	-.111E+00	-.135E+00	-.137E+00
.70	-.143E+00	-.168E+00	-.210E+00	-.231E+00	-.231E+00
.80	-.337E+00	-.360E+00	-.391E+00	-.393E+00	-.389E+00
.90	-.758E+00	-.743E+00	-.699E+00	-.653E+00	-.643E+00
.94	-.104E+01	-.984E+00	-.874E+00	-.795E+00	-.783E+00
.98	-.146E+01	-.132E+01	-.109E+01	-.971E+00	-.958E+00
1.00	-.176E+01	-.155E+01	-.123E+01	-.108E+01	-.107E+01
			$\sigma(x,t)/(N_0/\beta)$		
-1.0	.145E+01	.128E+01	.104E+01	.921E+00	.910E+00
-.98	.741E+00	.682E+00	.582E+00	.524E+00	.517E+00
-.94	.212E-01	.445E-01	.718E-01	.747E-01	.733E-01
-.90	-.185E+00	-.154E+00	-.107E+00	-.865E-01	-.854E-01
-.80	-.121E+00	-.115E+00	-.100E+00	-.879E-01	-.861E-01
-.70	-.442E-01	-.453E-01	-.450E-01	-.417E-01	-.407E-01
-.60	-.174E-01	-.195E-01	-.224E-01	-.226E-01	-.221E-01
-.40	-.311E-02	-.418E-02	-.634E-02	-.785E-02	-.792E-02
-.20	-.570E-03	-.916E-03	-.181E-02	-.285E-02	-.308E-02
0.00	-.196E-03	-.365E-03	-.878E-03	-.167E-02	-.193E-02
.20	-.570E-03	-.916E-03	-.181E-02	-.285E-02	-.308E-02
.40	-.311E-02	-.418E-02	-.634E-02	-.785E-02	-.792E-02
.60	-.174E-01	-.195E-01	-.224E-01	-.226E-01	-.221E-01
.70	-.442E-01	-.453E-01	-.450E-01	-.417E-01	-.407E-01
.80	-.121E+00	-.115E+00	-.100E+00	-.879E-01	-.861E-01
.90	-.185E+00	-.154E+00	-.107E+00	-.865E-01	-.854E-01
.94	.212E-01	.445E-01	.718E-01	.747E-01	.733E-01
.98	.741E+00	.682E+00	.582E+00	.524E+00	.517E+00
1.00	.145E+01	.128E+01	.104E+01	.921E+00	.910E+00

Table 5. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.140E+03	-.129E+03	-.110E+03	-.980E+02	-.964E+02
-.98	-.116E+03	-.109E+03	-.957E+02	-.865E+02	-.851E+02
-.94	-.830E+02	-.799E+02	-.737E+02	-.684E+02	-.673E+02
-.90	-.589E+02	-.580E+02	-.559E+02	-.532E+02	-.525E+02
-.80	-.199E+02	-.213E+02	-.236E+02	-.246E+02	-.245E+02
-.70	-.247E+01	-.380E+01	-.639E+01	-.829E+01	-.852E+01
-.60	.448E+01	.364E+01	.183E+01	.190E+00	-.123E+00
-.40	.826E+01	.803E+01	.744E+01	.672E+01	.649E+01
-.20	.884E+01	.879E+01	.864E+01	.840E+01	.828E+01
0.00	.893E+01	.892E+01	.889E+01	.881E+01	.877E+01
.20	.895E+01	.894E+01	.894E+01	.892E+01	.890E+01
.40	.895E+01	.895E+01	.895E+01	.895E+01	.894E+01
.60	.897E+01	.897E+01	.898E+01	.897E+01	.897E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.900E+01
.80	.910E+01	.909E+01	.907E+01	.905E+01	.904E+01
.90	.903E+01	.901E+01	.897E+01	.895E+01	.895E+01
.94	.875E+01	.875E+01	.876E+01	.877E+01	.877E+01
.98	.831E+01	.836E+01	.846E+01	.852E+01	.853E+01
1.00	.819E+01	.826E+01	.838E+01	.846E+01	.847E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	-.207E+03	-.195E+03	-.174E+03	-.162E+03	-.161E+03
-.98	-.879E+02	-.847E+02	-.787E+02	-.742E+02	-.735E+02
-.94	.120E+02	.999E+01	.683E+01	.543E+01	.542E+01
-.90	.295E+02	.278E+02	.246E+02	.226E+02	.224E+02
-.80	.124E+02	.122E+02	.116E+02	.109E+02	.108E+02
-.70	.384E+01	.392E+01	.399E+01	.390E+01	.384E+01
-.60	.142E+01	.154E+01	.175E+01	.186E+01	.185E+01
-.40	.221E+00	.273E+00	.386E+00	.493E+00	.511E+00
-.20	.340E-01	.474E-01	.812E-01	.125E+00	.139E+00
0.00	.526E-02	.820E-02	.167E-01	.309E-01	.376E-01
.20	.986E-03	.163E-02	.369E-02	.796E-02	.105E-01
.40	.130E-02	.155E-02	.220E-02	.334E-02	.426E-02
.60	.700E-02	.664E-02	.566E-02	.464E-02	.468E-02
.70	.301E-01	.302E-01	.301E-01	.300E-01	.302E-01
.80	.203E+00	.213E+00	.231E+00	.243E+00	.245E+00
.90	.259E+00	.240E+00	.205E+00	.185E+00	.184E+00
.94	-.136E+01	-.143E+01	-.154E+01	-.160E+01	-.161E+01
.98	-.753E+01	-.751E+01	-.747E+01	-.744E+01	-.744E+01
1.00	-.142E+02	-.139E+02	-.135E+02	-.132E+02	-.132E+02

Table 6. Adhesive stresses for a cover plate subjected to transverse shear loading ($Q_0 \neq 0$, $N_0=M_0=\Delta T=0$) for T=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=2.54\times10^{-2}$m.

T=43°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.136E+03	-.124E+03	-.104E+03	-.918E+02	-.904E+02
-.98	-.113E+03	-.105E+03	-.911E+02	-.817E+02	-.804E+02
-.94	-.819E+02	-.783E+02	-.712E+02	-.655E+02	-.644E+02
-.90	-.586E+02	-.575E+02	-.548E+02	-.517E+02	-.509E+02
-.80	-.204E+02	-.219E+02	-.243E+02	-.251E+02	-.249E+02
-.70	-.294E+01	-.446E+01	-.732E+01	-.925E+01	-.941E+01
-.60	.419E+01	.319E+01	.111E+01	-.680E+00	-.973E+00
-.40	.818E+01	.789E+01	.716E+01	.630E+01	.604E+01
-.20	.882E+01	.875E+01	.855E+01	.824E+01	.810E+01
0.00	.893E+01	.891E+01	.886E+01	.876E+01	.870E+01
.20	.895E+01	.894E+01	.893E+01	.890E+01	.888E+01
.40	.895E+01	.895E+01	.895E+01	.894E+01	.893E+01
.60	.897E+01	.897E+01	.898E+01	.897E+01	.897E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.900E+01
.80	.909E+01	.908E+01	.906E+01	.904E+01	.904E+01
.90	.902E+01	.900E+01	.896E+01	.894E+01	.894E+01
.94	.875E+01	.875E+01	.876E+01	.877E+01	.878E+01
.98	.832E+01	.838E+01	.848E+01	.855E+01	.856E+01
1.00	.821E+01	.828E+01	.841E+01	.850E+01	.851E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	-.201E+03	-.187E+03	-.166E+03	-.154E+03	-.152E+03
-.98	-.866E+02	-.829E+02	-.762E+02	-.715E+02	-.708E+02
-.94	.107E+02	.862E+01	.542E+01	.422E+01	.425E+01
-.90	.287E+02	.267E+02	.233E+02	.213E+02	.211E+02
-.80	.124E+02	.121E+02	.114E+02	.106E+02	.105E+02
-.70	.388E+01	.396E+01	.400E+01	.385E+01	.378E+01
-.60	.146E+01	.159E+01	.181E+01	.190E+01	.188E+01
-.40	.238E+00	.300E+00	.433E+00	.548E+00	.564E+00
-.20	.384E-01	.553E-01	.978E-01	.151E+00	.165E+00
0.00	.622E-02	.101E-01	.216E-01	.404E-01	.486E-01
.20	.120E-02	.209E-02	.502E-02	.110E-01	.146E-01
.40	.140E-02	.172E-02	.260E-02	.430E-02	.567E-02
.60	.699E-02	.652E-02	.529E-02	.426E-02	.452E-02
.70	.311E-01	.313E-01	.314E-01	.317E-01	.321E-01
.80	.211E+00	.222E+00	.243E+00	.256E+00	.257E+00
.90	.239E+00	.216E+00	.177E+00	.156E+00	.154E+00
.94	-.141E+01	-.149E+01	-.161E+01	-.167E+01	-.167E+01
.98	-.751E+01	-.748E+01	-.743E+01	-.741E+01	-.740E+01
1.00	-.140E+02	-.137E+02	-.132E+02	-.130E+02	-.130E+02

Table 6. Continued

T=60°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.131E+03	-.118E+03	-.955E+02	-.837E+02	-.824E+02
-.98	-.110E+03	-.101E+03	-.849E+02	-.753E+02	-.741E+02
-.94	-.804E+02	-.761E+02	-.677E+02	-.614E+02	-.604E+02
-.90	-.582E+02	-.567E+02	-.531E+02	-.494E+02	-.486E+02
-.80	-.211E+02	-.227E+02	-.250E+02	-.254E+02	-.251E+02
-.70	-.355E+01	-.537E+01	-.861E+01	-.105E+02	-.105E+02
-.60	.380E+01	.255E+01	.349E-01	-.191E+01	-.215E+01
-.40	.807E+01	.767E+01	.671E+01	.561E+01	.534E+01
-.20	.880E+01	.870E+01	.840E+01	.795E+01	.777E+01
0.00	.892E+01	.890E+01	.882E+01	.866E+01	.857E+01
.20	.894E+01	.894E+01	.892E+01	.887E+01	.883E+01
.40	.895E+01	.895E+01	.895E+01	.893E+01	.892E+01
.60	.897E+01	.898E+01	.898E+01	.897E+01	.896E+01
.70	.901E+01	.901E+01	.901E+01	.900E+01	.899E+01
.80	.909E+01	.908E+01	.905E+01	.903E+01	.902E+01
.90	.901E+01	.898E+01	.894E+01	.893E+01	.893E+01
.94	.874E+01	.874E+01	.876E+01	.878E+01	.878E+01
.98	.833E+01	.840E+01	.852E+01	.859E+01	.860E+01
1.00	.823E+01	.831E+01	.846E+01	.855E+01	.855E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	-.193E+03	-.178E+03	-.154E+03	-.143E+03	-.142E+03
-.98	-.848E+02	-.805E+02	-.728E+02	-.679E+02	-.672E+02
-.94	.924E+01	.689E+01	.369E+01	.282E+01	.290E+01
-.90	.277E+02	.254E+02	.216E+02	.197E+02	.196E+02
-.80	.124E+02	.120E+02	.111E+02	.102E+02	.101E+02
-.70	.393E+01	.400E+01	.398E+01	.374E+01	.366E+01
-.60	.151E+01	.167E+01	.189E+01	.193E+01	.189E+01
-.40	.262E+00	.341E+00	.501E+00	.624E+00	.633E+00
-.20	.448E-01	.677E-01	.125E+00	.191E+00	.206E+00
0.00	.768E-02	.134E-01	.303E-01	.572E-01	.673E-01
.20	.153E-02	.290E-02	.757E-02	.171E-01	.223E-01
.40	.155E-02	.200E-02	.335E-02	.630E-02	.858E-02
.60	.694E-02	.628E-02	.472E-02	.392E-02	.468E-02
.70	.326E-01	.330E-01	.336E-01	.345E-01	.352E-01
.80	.221E+00	.235E+00	.259E+00	.272E+00	.274E+00
.90	.213E+00	.184E+00	.137E+00	.116E+00	.115E+00
.94	-.148E+01	-.157E+01	-.170E+01	-.175E+01	-.175E+01
.98	-.748E+01	-.744E+01	-.738E+01	-.735E+01	-.735E+01
1.00	-.137E+02	-.134E+02	-.128E+02	-.126E+02	-.126E+02

Table 6. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(Q_0/\beta)$		
-1.0	-.128E+03	-.112E+03	-.890E+02	-.775E+02	-.764E+02
-.98	-.108E+03	-.971E+02	-.799E+02	-.703E+02	-.692E+02
-.94	-.792E+02	-.741E+02	-.647E+02	-.581E+02	-.571E+02
-.90	-.578E+02	-.560E+02	-.516E+02	-.474E+02	-.466E+02
-.80	-.216E+02	-.234E+02	-.256E+02	-.256E+02	-.252E+02
-.70	-.403E+01	-.612E+01	-.964E+01	-.114E+02	-.113E+02
-.60	.350E+01	.202E+01	-.863E+00	-.287E+01	-.305E+01
-.40	.798E+01	.749E+01	.629E+01	.501E+01	.473E+01
-.20	.878E+01	.864E+01	.826E+01	.767E+01	.747E+01
0.00	.892E+01	.889E+01	.877E+01	.855E+01	.843E+01
.20	.894E+01	.894E+01	.891E+01	.883E+01	.877E+01
.40	.895E+01	.895E+01	.895E+01	.892E+01	.889E+01
.60	.898E+01	.898E+01	.898E+01	.897E+01	.895E+01
.70	.902E+01	.902E+01	.901E+01	.899E+01	.898E+01
.80	.910E+01	.908E+01	.905E+01	.902E+01	.901E+01
.90	.900E+01	.897E+01	.893E+01	.892E+01	.892E+01
.94	.873E+01	.873E+01	.875E+01	.878E+01	.878E+01
.98	.833E+01	.841E+01	.854E+01	.861E+01	.861E+01
1.00	.823E+01	.832E+01	.849E+01	.857E+01	.857E+01
			$\sigma(x,t)/(Q_0/\beta)$		
-1.0	-.183E+03	-.166E+03	-.142E+03	-.131E+03	-.130E+03
-.98	-.829E+02	-.778E+02	-.693E+02	-.643E+02	-.637E+02
-.94	.713E+01	.456E+01	.141E+01	.849E+00	.957E+00
-.90	.264E+02	.238E+02	.197E+02	.179E+02	.178E+02
-.80	.126E+02	.121E+02	.110E+02	.101E+02	.993E+01
-.70	.401E+01	.407E+01	.399E+01	.369E+01	.360E+01
-.60	.155E+01	.172E+01	.193E+01	.192E+01	.188E+01
-.40	.280E+00	.374E+00	.558E+00	.681E+00	.684E+00
-.20	.499E-01	.787E-01	.150E+00	.227E+00	.241E+00
0.00	.889E-02	.164E-01	.391E-01	.736E-01	.847E-01
.20	.182E-02	.372E-02	.103E-01	.237E-01	.302E-01
.40	.171E-02	.232E-02	.424E-02	.870E-02	.119E-01
.60	.713E-02	.629E-02	.445E-02	.409E-02	.540E-02
.70	.363E-01	.375E-01	.397E-01	.419E-01	.431E-01
.80	.237E+00	.255E+00	.283E+00	.298E+00	.299E+00
.90	.162E+00	.122E+00	.609E-01	.364E-01	.359E-01
.94	-.159E+01	-.169E+01	-.183E+01	-.188E+01	-.188E+01
.98	-.741E+01	-.736E+01	-.728E+01	-.725E+01	-.725E+01
1.00	-.133E+02	-.128E+02	-.123E+02	-.121E+02	-.121E+02

Table 6. Continued

T=21°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(\Delta T/\beta)$		
-1.0	.344E+02	.317E+02	.271E+02	.241E+02	.238E+02
-.98	.279E+02	.263E+02	.232E+02	.210E+02	.207E+02
-.94	.193E+02	.187E+02	.174E+02	.163E+02	.161E+02
-.90	.137E+02	.136E+02	.133E+02	.128E+02	.126E+02
-.80	.565E+01	.596E+01	.649E+01	.675E+01	.674E+01
-.70	.224E+01	.252E+01	.306E+01	.347E+01	.352E+01
-.60	.876E+00	.105E+01	.142E+01	.176E+01	.183E+01
-.40	.135E+00	.182E+00	.300E+00	.446E+00	.493E+00
-.20	.202E-01	.306E-01	.595E-01	.105E+00	.124E+00
0.00	.975E-11	-.143E-10	-.478E-11	-.215E-11	-.187E-11
.20	-.202E-01	-.306E-01	-.595E-01	-.105E+00	-.124E+00
.40	-.135E+00	-.182E+00	-.300E+00	-.446E+00	-.493E+00
.60	-.876E+00	-.105E+01	-.142E+01	-.176E+01	-.183E+01
.70	-.224E+01	-.252E+01	-.306E+01	-.347E+01	-.352E+01
.80	-.565E+01	-.596E+01	-.649E+01	-.675E+01	-.674E+01
.90	-.137E+02	-.136E+02	-.133E+02	-.128E+02	-.126E+02
.94	-.193E+02	-.187E+02	-.174E+02	-.163E+02	-.161E+02
.98	-.279E+02	-.263E+02	-.232E+02	-.210E+02	-.207E+02
1.00	-.344E+02	-.317E+02	-.271E+02	-.241E+02	-.238E+02
			$\sigma(x,t)/(\Delta T/\beta)$		
-1.0	.296E+02	.274E+02	.237E+02	.214E+02	.211E+02
-.98	.142E+02	.135E+02	.122E+02	.113E+02	.111E+02
-.94	-.325E+00	.166E-01	.552E+00	.776E+00	.773E+00
-.90	-.388E+01	-.351E+01	-.284E+01	-.241E+01	-.236E+01
-.80	-.218E+01	-.212E+01	-.198E+01	-.183E+01	-.179E+01
-.70	-.753E+00	-.774E+00	-.799E+00	-.786E+00	-.773E+00
-.60	-.281E+00	-.308E+00	-.355E+00	-.381E+00	-.379E+00
-.40	-.434E-01	-.540E-01	-.773E-01	-.996E-01	-.104E+00
-.20	-.684E-02	-.964E-02	-.169E-01	-.266E-01	-.300E-01
0.00	-.206E-02	-.323E-02	-.665E-02	-.124E-01	-.151E-01
.20	-.684E-02	-.964E-02	-.169E-01	-.266E-01	-.300E-01
.40	-.434E-01	-.540E-01	-.773E-01	-.996E-01	-.104E+00
.60	-.281E+00	-.308E+00	-.355E+00	-.381E+00	-.379E+00
.70	-.753E+00	-.774E+00	-.799E+00	-.786E+00	-.773E+00
.80	-.218E+01	-.212E+01	-.198E+01	-.183E+01	-.179E+01
.90	-.388E+01	-.351E+01	-.284E+01	-.241E+01	-.236E+01
.94	-.325E+00	.166E-01	.552E+00	.776E+00	.773E+00
.98	.142E+02	.135E+02	.122E+02	.113E+02	.111E+02
1.00	.296E+02	.274E+02	.237E+02	.214E+02	.211E+02

Table 7. Adhesive stresses for a cover plate resulting from a temperature increase ($\Delta T \neq 0$, $N_0=M_0=Q_0=0$) for T_f=21°C, 43°C, 60°C, and 82°C, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and $\beta=(2.54\times10^{-2}\text{m})^2(5/9°\text{C})/(4.448\text{N})$.

T=43°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(\Delta T/\beta)$		
-1.0	.334E+02	.305E+02	.256E+02	.227E+02	.223E+02
-.98	.273E+02	.255E+02	.221E+02	.199E+02	.197E+02
-.94	.191E+02	.184E+02	.169E+02	.157E+02	.155E+02
-.90	.136E+02	.135E+02	.131E+02	.125E+02	.123E+02
-.80	.576E+01	.610E+01	.665E+01	.686E+01	.683E+01
-.70	.233E+01	.265E+01	.326E+01	.367E+01	.371E+01
-.60	.936E+00	.114E+01	.157E+01	.195E+01	.201E+01
-.40	.151E+00	.210E+00	.356E+00	.533E+00	.585E+00
-.20	.236E-01	.371E-01	.754E-01	.134E+00	.158E+00
0.00	-.856E-12	-.593E-12	-.466E-12	-.425E-12	-.731E-12
.20	-.236E-01	-.371E-01	-.754E-01	-.134E+00	-.158E+00
.40	-.151E+00	-.210E+00	-.356E+00	-.533E+00	-.585E+00
.60	-.936E+00	-.114E+01	-.157E+01	-.195E+01	-.201E+01
.70	-.233E+01	-.265E+01	-.326E+01	-.367E+01	-.371E+01
.80	-.576E+01	-.610E+01	-.665E+01	-.686E+01	-.683E+01
.90	-.136E+02	-.135E+02	-.131E+02	-.125E+02	-.123E+02
.94	-.191E+02	-.184E+02	-.169E+02	-.157E+02	-.155E+02
.98	-.273E+02	-.255E+02	-.221E+02	-.199E+02	-.197E+02
1.00	-.334E+02	-.305E+02	-.256E+02	-.227E+02	-.223E+02
			$\sigma(x,t)/(\Delta T/\beta)$		
-1.0	.285E+02	.262E+02	.222E+02	.200E+02	.197E+02
-.98	.139E+02	.132E+02	.117E+02	.107E+02	.106E+02
-.94	-.137E+00	.225E+00	.758E+00	.940E+00	.929E+00
-.90	-.372E+01	-.330E+01	-.259E+01	-.216E+01	-.212E+01
-.80	-.217E+01	-.210E+01	-.193E+01	-.175E+01	-.172E+01
-.70	-.763E+00	-.784E+00	-.802E+00	-.777E+00	-.763E+00
-.60	-.290E+00	-.320E+00	-.370E+00	-.391E+00	-.387E+00
-.40	-.470E-01	-.596E-01	-.869E-01	-.111E+00	-.115E+00
-.20	-.776E-02	-.113E-01	-.205E-01	-.325E-01	-.363E-01
0.00	-.243E-02	-.399E-02	-.861E-02	-.162E-01	-.196E-01
.20	-.776E-02	-.113E-01	-.205E-01	-.325E-01	-.363E-01
.40	-.470E-01	-.596E-01	-.869E-01	-.111E+00	-.115E+00
.60	-.290E+00	-.320E+00	-.370E+00	-.391E+00	-.387E+00
.70	-.763E+00	-.784E+00	-.802E+00	-.777E+00	-.763E+00
.80	-.217E+01	-.210E+01	-.193E+01	-.175E+01	-.172E+01
.90	-.372E+01	-.330E+01	-.259E+01	-.216E+01	-.212E+01
.94	-.137E+00	.225E+00	.758E+00	.940E+00	.929E+00
.98	.139E+02	.132E+02	.117E+02	.107E+02	.106E+02
1.00	.285E+02	.262E+02	.222E+02	.200E+02	.197E+02

Table 7. Continued

T=60°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(\Delta T/\beta)$		
-1.0	.323E+02	.289E+02	.235E+02	.207E+02	.204E+02
-.98	.266E+02	.244E+02	.207E+02	.185E+02	.182E+02
-.94	.188E+02	.179E+02	.162E+02	.149E+02	.147E+02
-.90	.136E+02	.134E+02	.128E+02	.120E+02	.119E+02
-.80	.590E+01	.628E+01	.684E+01	.697E+01	.691E+01
-.70	.246E+01	.284E+01	.353E+01	.394E+01	.395E+01
-.60	.102E+01	.127E+01	.179E+01	.220E+01	.225E+01
-.40	.173E+00	.253E+00	.449E+00	.672E+00	.726E+00
-.20	.287E-01	.480E-01	.103E+00	.185E+00	.214E+00
0.00	-.146E-08	.109E-08	-.542E-09	-.226E-09	-.482E-10
.20	-.287E-01	-.480E-01	-.103E+00	-.185E+00	-.214E+00
.40	-.173E+00	-.253E+00	-.449E+00	-.672E+00	-.726E+00
.60	-.102E+01	-.127E+01	-.179E+01	-.220E+01	-.225E+01
.70	-.246E+01	-.284E+01	-.353E+01	-.394E+01	-.395E+01
.80	-.590E+01	-.628E+01	-.684E+01	-.697E+01	-.691E+01
.90	-.136E+02	-.134E+02	-.128E+02	-.120E+02	-.119E+02
.94	-.188E+02	-.179E+02	-.162E+02	-.149E+02	-.147E+02
.98	-.266E+02	-.244E+02	-.207E+02	-.185E+02	-.182E+02
1.00	-.323E+02	-.289E+02	-.235E+02	-.207E+02	-.204E+02
			$\sigma(x,t)/(\Delta T/\beta)$		
-1.0	.273E+02	.246E+02	.204E+02	.182E+02	.180E+02
-.98	.136E+02	.127E+02	.110E+02	.100E+02	.987E+01
-.94	.894E-01	.483E+00	.100E+01	.112E+01	.110E+01
-.90	-.352E+01	-.303E+01	-.225E+01	-.185E+01	-.182E+01
-.80	-.216E+01	-.206E+01	-.184E+01	-.164E+01	-.161E+01
-.70	-.775E+00	-.795E+00	-.801E+00	-.757E+00	-.741E+00
-.60	-.302E+00	-.336E+00	-.388E+00	-.400E+00	-.393E+00
-.40	-.518E-01	-.679E-01	-.101E+00	-.128E+00	-.130E+00
-.20	-.910E-02	-.140E-01	-.266E-01	-.422E-01	-.464E-01
0.00	-.301E-02	-.529E-02	-.121E-01	-.231E-01	-.273E-01
.20	-.910E-02	-.140E-01	-.266E-01	-.422E-01	-.464E-01
.40	-.518E-01	-.679E-01	-.101E+00	-.128E+00	-.130E+00
.60	-.302E+00	-.336E+00	-.388E+00	-.400E+00	-.393E+00
.70	-.775E+00	-.795E+00	-.801E+00	-.757E+00	-.741E+00
.80	-.216E+01	-.206E+01	-.184E+01	-.164E+01	-.161E+01
.90	-.352E+01	-.303E+01	-.225E+01	-.185E+01	-.182E+01
.94	.894E-01	.483E+00	.100E+01	.112E+01	.110E+01
.98	.136E+02	.127E+02	.110E+02	.100E+02	.987E+01
1.00	.273E+02	.246E+02	.204E+02	.182E+02	.180E+02

Table 7. Continued

T=82°C

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
			$\tau(x,t)/(\Delta T/\beta)$		
-1.0	.315E+02	.277E+02	.220E+02	.193E+02	.190E+02
-.98	.260E+02	.236E+02	.195E+02	.173E+02	.171E+02
-.94	.186E+02	.176E+02	.156E+02	.142E+02	.140E+02
-.90	.135E+02	.133E+02	.125E+02	.117E+02	.115E+02
-.80	.601E+01	.643E+01	.697E+01	.701E+01	.694E+01
-.70	.256E+01	.300E+01	.375E+01	.413E+01	.412E+01
-.60	.108E+01	.138E+01	.198E+01	.240E+01	.244E+01
-.40	.191E+00	.291E+00	.533E+00	.793E+00	.846E+00
-.20	.329E-01	.582E-01	.131E+00	.234E+00	.265E+00
0.00	-.393E-09	-.371E-10	-.131E-11	.316E-13	-.160E-11
.20	-.329E-01	-.582E-01	-.131E+00	-.234E+00	-.265E+00
.40	-.191E+00	-.291E+00	-.533E+00	-.793E+00	-.846E+00
.60	-.108E+01	-.138E+01	-.198E+01	-.240E+01	-.244E+01
.70	-.256E+01	-.300E+01	-.375E+01	-.413E+01	-.412E+01
.80	-.601E+01	-.643E+01	-.697E+01	-.701E+01	-.694E+01
.90	-.135E+02	-.133E+02	-.125E+02	-.117E+02	-.115E+02
.94	-.186E+02	-.176E+02	-.156E+02	-.142E+02	-.140E+02
.98	-.260E+02	-.236E+02	-.195E+02	-.173E+02	-.171E+02
1.00	-.315E+02	-.277E+02	-.220E+02	-.193E+02	-.190E+02
			$\sigma(x,t)/(\Delta T/\beta)$		
-1.0	.258E+02	.229E+02	.185E+02	.164E+02	.162E+02
-.98	.132E+02	.122E+02	.104E+02	.935E+01	.922E+01
-.94	.378E+00	.794E+00	.128E+01	.133E+01	.131E+01
-.90	-.330E+01	-.275E+01	-.191E+01	-.154E+01	-.152E+01
-.80	-.217E+01	-.205E+01	-.179E+01	-.157E+01	-.154E+01
-.70	-.789E+00	-.809E+00	-.803E+00	-.744E+00	-.726E+00
-.60	-.311E+00	-.348E+00	-.400E+00	-.403E+00	-.394E+00
-.40	-.555E-01	-.747E-01	-.113E+00	-.140E+00	-.141E+00
-.20	-.102E-01	-.164E-01	-.322E-01	-.508E-01	-.550E-01
0.00	-.350E-02	-.652E-02	-.157E-01	-.298E-01	-.344E-01
.20	-.102E-01	-.164E-01	-.322E-01	-.508E-01	-.550E-01
.40	-.555E-01	-.747E-01	-.113E+00	-.140E+00	-.141E+00
.60	-.311E+00	-.348E+00	-.400E+00	-.403E+00	-.394E+00
.70	-.789E+00	-.809E+00	-.803E+00	-.744E+00	-.726E+00
.80	-.217E+01	-.205E+01	-.179E+01	-.157E+01	-.154E+01
.90	-.330E+01	-.275E+01	-.191E+01	-.154E+01	-.152E+01
.94	.378E+00	.794E+00	.128E+01	.133E+01	.131E+01
.98	.132E+02	.122E+02	.104E+02	.935E+01	.922E+01
1.00	.258E+02	.229E+02	.185E+02	.164E+02	.162E+02

Table 7. Continued

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\tau(x,t)/(M_0/\beta^2)$ ℓ = 2.54 mm			
-1.0	.488E+03	.438E+03	.351E+03	.301E+03	.296E+03
-.98	.259E+03	.243E+03	.212E+03	.189E+03	.186E+03
-.94	.482E+02	.541E+02	.629E+02	.650E+02	.642E+02
-.90	-.901E+01	-.237E+01	.965E+01	.169E+02	.174E+02
-.80	-.130E+02	-.116E+02	-.835E+01	-.490E+01	-.412E+01
-.70	-.415E+01	-.407E+01	-.369E+01	-.289E+01	-.253E+01
-.60	-.113E+01	-.118E+01	-.122E+01	-.111E+01	-.981E+00
-.40	-.772E-01	-.879E-01	-.111E+00	-.127E+00	-.118E+00
-.20	-.518E-02	-.641E-02	-.951E-02	-.134E-01	-.137E-01
0.00	-.510E-03	-.711E-03	-.133E-02	-.261E-02	-.362E-02
.20	-.250E-02	-.352E-02	-.670E-02	-.134E-01	-.190E-01
.40	-.371E-01	-.484E-01	-.803E-01	-.135E+00	-.168E+00
.60	-.556E+00	-.670E+00	-.959E+00	-.134E+01	-.149E+01
.70	-.216E+01	-.250E+01	-.329E+01	-.418E+01	-.444E+01
.80	-.848E+01	-.937E+01	-.112E+02	-.129E+02	-.132E+02
.90	-.344E+02	-.357E+02	-.379E+02	-.387E+02	-.385E+02
.94	-.617E+02	-.618E+02	-.612E+02	-.595E+02	-.588E+02
.98	-.114E+03	-.109E+03	-.984E+02	-.908E+02	-.895E+02
1.00	-.159E+03	-.147E+03	-.126E+03	-.113E+03	-.111E+03
		$\tau(x,t)/(M_0/\beta^2)$ ℓ = 254 mm			
-1.0	.488E+03	.438E+03	.351E+03	.301E+03	.296E+03
-.98	-.901E+01	-.237E+01	.965E+01	.159E+02	.174E+02
-.94	-.415E+01	-.407E+01	-.369E+01	-.289E+01	-.253E+01
-.90	-.297E+00	-.324E+00	-.373E+00	-.382E+00	-.345E+00
-.80	-.345E-03	-.461E-03	-.791E-03	-.133E-02	-.153E-02
-.70	-.397E-06	-.637E-06	-.150E-05	-.386E-05	-.643E-05
-.60	-.430E-09	-.848E-09	-.267E-08	-.100E-07	-.258E-07
-.40	.161E-10	.126E-10	.139E-10	.829E-11	.554E-11
-.20	.161E-10	.126E-10	.140E-10	.834E-11	.589E-11
0.00	.161E-10	.126E-10	.140E-10	.834E-11	.589E-11
.20	.161E-10	.126E-10	.140E-10	.834E-11	.589E-11
.40	.161E-10	.126E-10	.139E-10	.830E-11	.551E-11
.60	-.182E-09	-.432E-09	-.167E-08	-.817E-08	-.315E-07
.70	-.187E-06	-.340E-06	-.980E-06	-.336E-05	-.836E-05
.80	-.165E-03	-.250E-03	-.540E-03	-.127E-02	-.209E-02
.90	-.144E+00	-.180E+00	-.278E+00	-.428E+00	-.502E+00
.94	-.216E+01	-.250E+01	-.329E+01	-.418E+01	-.444E+01
.98	-.344E+02	-.357E+02	-.379E+02	-.387E+02	-.385E+02
1.00	-.159E+03	-.147E+03	-.126E+03	-.113E+03	-.111E+03

Table 8. Comparison of shear stress and normal stress for a single lap joint subjected to bending ($M_0 \neq 0$, $Q_0 = N_0 = \Delta T = 0$) for ℓ=25.4mm and 254.mm, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, T=21°C, and $\beta = 2.54 \times 10^{-2}$m.

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.

$\sigma(x,t)/(M_0/\beta^2)$ ℓ = 25.4 mm

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.149E+04	-.145E+04	-.140E+04	-.137E+04	-.137E+04
-.98	-.523E+01	-.138E+02	-.285E+02	-.371E+02	-.379E+02
-.94	.106E+03	.108E+03	.111E+03	.114E+03	.114E+03
-.90	.488E+02	.499E+02	.520E+02	.537E+02	.540E+02
-.80	.813E+01	.796E+01	.766E+01	.751E+01	.752E+01
-.70	.176E+01	.168E+01	.151E+01	.133E+01	.130E+01
-.60	.431E+00	.416E+00	.375E+00	.316E+00	.296E+00
-.40	.282E-01	.292E-01	.306E-01	.290E-01	.262E-01
-.20	.188E-02	.211E-02	.263E-02	.302E-02	.283E-02
0.00	.655E-04	.697E-04	.671E-04	.264E-06	-.128E-03
.20	-.892E-03	-.115E-02	-.189E-02	-.319E-02	-.401E-02
.40	-.135E-01	-.161E-01	-.227E-01	-.320E-01	-.361E-01
.60	-.201E+00	-.221E+00	-.265E+00	-.311E+00	-.321E+00
.70	-.774E+00	-.815E+00	-.897E+00	-.959E+00	-.965E+00
.80	-.294E+01	-.297E+01	-.300E+01	-.297E+01	-.295E+01
.90	-.107E+02	-.104E+02	-.994E+01	-.948E+01	-.939E+01
.94	-.165E+02	-.159E+02	-.149E+02	-.142E+02	-.141E+02
.98	.127E+02	.138E+02	.155E+02	.160E+02	.159E+02
1.00	.243E+03	.231E+03	.211E+03	.199E+03	.198E+03

$\sigma(x,t)/(M_0/\beta^2)$ ℓ = 254 mm

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.149E+04	-.145E+04	-.140E+04	-.137E+04	-.137E+04
-.98	.488E+02	.499E+02	.520E+02	.537E+02	.540E+02
-.94	.176E+01	.168E+01	.151E+01	.133E+01	.130E+01
-.90	.109E+00	.109E+00	.105E+00	.917E-01	.835E-01
-.80	.126E-03	.153E-03	.225E-03	.316E-03	.325E-03
-.70	.145E-06	.214E-06	.442E-06	.967E-06	.139E-05
-.60	.165E-09	.292E-09	.809E-09	.261E-08	.567E-08
-.40	.158E-15	.473E-15	.232E-14	.148E-13	.793E-13
-.20	-.933E-21	-.280E-21	.503E-20	.671E-19	.873E-18
0.00	.100E-25	.106E-25	.162E-25	.949E-25	.495E-24
.20	.162E-20	.127E-20	-.196E-20	-.472E-19	-.881E-18
.40	-.118E-16	-.190E-15	-.137E-14	-.112E-13	-.881E-13
.60	-.756E-10	-.153E-09	-.516E-09	-.218E-08	-.709E-08
.70	-.687E-07	-.115E-06	-.293E-06	-.869E-06	-.184E-05
.80	-.601E-04	-.837E-04	-.157E-03	-.315E-03	-.453E-03
.90	-.521E-01	-.596E-01	-.778E-01	-.100E+00	-.108E+00
.94	-.774E+00	-.815E+00	-.897E+00	-.959E+00	-.965E+00
.98	-.107E+02	-.104E+02	-.994E+01	-.948E+01	-.939E+01
1.00	.243E+03	.231E+03	.211E+03	.199E+03	.198E+03

Table 8. Continued

(x+ℓ)cm	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
	$\tau(x,t)/(M_0/\beta^2)$		ℓ = 20 mm		
0.00	.499E+03	.439E+03	.351E+03	.301E+03	.296E+03
.10	.126E+03	.126E+03	.122E+03	.115E+03	.113E+03
.20	.112E+02	.185E+02	.307E+02	.366E+02	.365E+02
.30	-.162E+02	-.106E+02	-.178E+00	.704E+01	.773E+01
.40	-.177E+02	-.146E+02	-.819E+01	-.267E+01	-.179E+01
.50	-.133E+02	-.119E+02	-.846E+01	-.487E+01	-.408E+01
.60	-.890E+01	-.829E+01	-.665E+01	-.452E+01	-.389E+01
.70	-.561E+01	-.541E+01	-.471E+01	-.352E+01	-.306E+01
.80	-.343E+01	-.340E+01	-.316E+01	-.253E+01	-.222E+01
.90	-.207E+01	-.210E+01	-.205E+01	-.175E+01	-.154E+01
1.00	-.123E+01	-.128E+01	-.131E+01	-.118E+01	-.105E+01
1.10	-.729E+00	-.771E+00	-.829E+00	-.784E+00	-.699E+00
1.20	-.431E+00	-.464E+00	-.519E+00	-.515E+00	-.463E+00
1.30	-.254E+00	-.278E+00	-.324E+00	-.336E+00	-.306E+00
1.40	-.149E+00	-.167E+00	-.201E+00	-.219E+00	-.201E+00
1.50	-.879E-01	-.998E-01	-.125E+00	-.142E+00	-.133E+00
1.60	-.519E-01	-.600E-01	-.779E-01	-.930E-01	-.892E-01
1.70	-.309E-01	-.364E-01	-.492E-01	-.621E-01	-.616E-01
1.80	-.188E-01	-.226E-01	-.321E-01	-.435E-01	-.455E-01
	$\tau(x,t)/(M_0/\beta^2)$		ℓ = 100 mm		
0.00	.488E+03	.439E+03	.351E+03	.301E+03	.296E+03
.10	.126E+03	.126E+03	.122E+03	.115E+03	.113E+03
.20	.112E+02	.185E+02	.307E+02	.366E+02	.365E+02
.30	-.162E+02	-.106E+02	-.178E+00	.704E+01	.773E+01
.40	-.177E+02	-.146E+02	-.819E+01	-.267E+01	-.179E+01
.50	-.133E+02	-.119E+02	-.846E+01	-.487E+01	-.408E+01
.60	-.890E+01	-.829E+01	-.665E+01	-.452E+01	-.389E+01
.70	-.561E+01	-.541E+01	-.471E+01	-.352E+01	-.306E+01
.80	-.343E+01	-.340E+01	-.316E+01	-.253E+01	-.222E+01
.90	-.207E+01	-.210E+01	-.205E+01	-.175E+01	-.154E+01
1.00	-.123E+01	-.128E+01	-.131E+01	-.118E+01	-.105E+01
1.10	-.729E+00	-.771E+00	-.829E+00	-.783E+00	-.699E+00
1.20	-.431E+00	-.464E+00	-.519E+00	-.515E+00	-.462E+00
1.30	-.254E+00	-.278E+00	-.324E+00	-.335E+00	-.304E+00
1.40	-.149E+00	-.166E+00	-.201E+00	-.218E+00	-.200E+00
1.50	-.877E-01	-.995E-01	-.124E+00	-.141E+00	-.131E+00
1.60	-.516E-01	-.594E-01	-.769E-01	-.906E-01	-.854E-01
1.70	-.303E-01	-.355E-01	-.474E-01	-.582E-01	-.557E-01
1.80	-.178E-01	-.212E-01	-.292E-01	-.373E-01	-.364E-01

Table 9. Comparison of shear stress and normal stress near $x=-\ell$ for a single lap joint subjected to bending ($M_0 \neq 0$, $Q_0 = N_0 = \Delta T = 0$) for ℓ=20mm and 100mm, where h_1=.762mm, h_2=2.286mm, h_0=.1016mm, T=21°C, and $\beta = 2.54 \times 10^{-2}$m.

(x+ℓ)cm	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\sigma(x,t)/(M_0/\beta^2)$	ℓ = 20 mm		
0.00	-.149E+04	-.145E+04	-.140E+04	-.137E+04	-.137E+04
.10	.132E+03	.132E+03	.131E+03	.130E+03	.130E+03
.20	.742E+02	.760E+02	.793E+02	.817E+02	.821E+02
.30	.344E+02	.350E+02	.361E+02	.372E+02	.375E+02
.40	.167E+02	.167E+02	.168E+02	.170E+02	.171E+02
.50	.856E+01	.839E+01	.810E+01	.796E+01	.798E+01
.60	.457E+01	.442E+01	.411E+01	.388E+01	.386E+01
.70	.252E+01	.241E+01	.218E+01	.198E+01	.194E+01
.80	.142E+01	.136E+01	.121E+01	.106E+01	.102E+01
.90	.813E+00	.779E+00	.693E+00	.590E+00	.563E+00
1.00	.470E+00	.453E+00	.408E+00	.344E+00	.322E+00
1.10	.273E+00	.266E+00	.244E+00	.207E+00	.191E+00
1.20	.159E+00	.157E+00	.148E+00	.127E+00	.116E+00
1.30	.932E-01	.934E-01	.906E-01	.797E-01	.723E-01
1.40	.545E-01	.555E-01	.556E-01	.503E-01	.454E-01
1.50	.319E-01	.330E-01	.342E-01	.319E-01	.287E-01
1.60	.186E-01	.195E-01	.209E-01	.201E-01	.179E-01
1.70	.108E-01	.115E-01	.126E-01	.123E-01	.108E-01
1.80	.610E-02	.653E-02	.725E-02	.702E-02	.587E-02
		$\sigma(x,t)/(M_0/\beta^2)$	ℓ = 100 mm		
0.00	-.149E+04	-.145E+04	-.140E+04	-.137E+04	-.137E+04
.10	.132E+03	.132E+03	.131E+03	.130E+03	.130E+03
.20	.742E+02	.760E+02	.793E+02	.817E+02	.821E+02
.30	.344E+02	.350E+02	.361E+02	.372E+02	.375E+02
.40	.167E+02	.167E+02	.168E+02	.170E+02	.171E+02
.50	.856E+01	.839E+01	.810E+01	.796E+01	.798E+01
.60	.457E+01	.442E+01	.411E+01	.388E+01	.386E+01
.70	.252E+01	.241E+01	.218E+01	.198E+01	.194E+01
.80	.142E+01	.136E+01	.121E+01	.106E+01	.102E+01
.90	.813E+00	.779E+00	.693E+00	.590E+00	.563E+00
1.00	.470E+00	.453E+00	.408E+00	.344E+00	.323E+00
1.10	.273E+00	.266E+00	.244E+00	.207E+00	.191E+00
1.20	.159E+00	.157E+00	.148E+00	.127E+00	.117E+00
1.30	.932E-01	.934E-01	.907E-01	.798E-01	.725E-01
1.40	.546E-01	.555E-01	.558E-01	.506E-01	.458E-01
1.50	.320E-01	.331E-01	.344E-01	.323E-01	.292E-01
1.60	.188E-01	.197E-01	.212E-01	.207E-01	.188E-01
1.70	.110E-01	.118E-01	.131E-01	.133E-01	.121E-01
1.80	.647E-02	.702E-02	.809E-02	.852E-02	.784E-02

Table 9. Continued

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\tau(x,t)/(M_0\beta^2)$ $h_0 = 2.43\times10^{-2}$ mm			
-1.0	.112E+04	.101E+04	.818E+03	.706E+03	.694E+03
-.98	.651E+03	.612E+03	.537E+03	.480E+03	.472E+03
-.94	.174E+03	.184E+03	.198E+03	.198E+03	.195E+03
-.90	.903E+00	.163E+02	.438E+02	.598E+02	.605E+02
-.80	-.571E+02	-.526E+02	-.425E+02	-.321E+02	-.298E+02
-.70	-.328E+02	-.323E+02	-.307E+02	-.280E+02	-.268E+02
-.60	-.159E+02	-.161E+02	-.162E+02	-.159E+02	-.155E+02
-.40	-.349E+01	-.358E+01	-.378E+01	-.396E+01	-.398E+01
-.20	-.765E+00	-.793E+00	-.857E+00	-.930E+00	-.952E+00
0.00	-.207E+00	-.218E+00	-.243E+00	-.275E+00	-.289E+00
.20	-.237E+00	-.252E+00	-.289E+00	-.339E+00	-.362E+00
.40	-.940E+00	-.994E+00	-.112E+01	-.129E+01	-.137E+01
.60	-.434E+01	-.457E+01	-.511E+01	-.579E+01	-.605E+01
.70	-.955E+01	-.101E+02	-.113E+02	-.127E+02	-.131E+02
.80	-.224E+02	-.238E+02	-.267E+02	-.293E+02	-.298E+02
.90	-.657E+02	-.683E+02	-.725E+02	-.744E+02	-.741E+02
.94	-.114E+03	-.114E+03	-.114E+03	-.112E+03	-.111E+03
.98	-.214E+03	-.204E+03	-.186E+03	-.172E+03	-.169E+03
1.00	-.304E+03	-.281E+03	-.241E+03	-.216E+03	-.213E+03
		$\tau(x,t)/(M_0\beta^2)$ $h_0 = 5.08\times10^{-2}$ mm			
-1.0	.743E+03	.669E+03	.540E+03	.464E+03	.456E+03
-.98	.498E+03	.461E+03	.393E+03	.346E+03	.340E+03
-.94	.195E+03	.195E+03	.191E+03	.181E+03	.177E+03
-.90	.527E+02	.628E+02	.787E+02	.845E+02	.837E+02
-.80	-.354E+02	-.286E+02	-.150E+02	-.440E+01	-.299E+01
-.70	-.286E+02	-.266E+02	-.217E+02	-.163E+02	-.150E+02
-.60	-.160E+02	-.156E+02	-.144E+02	-.125E+02	-.117E+02
-.40	-.394E+01	-.406E+01	-.423E+01	-.420E+01	-.406E+01
-.20	-.927E+00	-.981E+00	-.110E+01	-.120E+01	-.120E+01
0.00	-.276E+00	-.303E+00	-.367E+00	-.450E+00	-.481E+00
.20	-.341E+00	-.384E+00	-.492E+00	-.644E+00	-.716E+00
.40	-.130E+01	-.144E+01	-.179E+01	-.224E+01	-.242E+01
.60	-.576E+01	-.628E+01	-.748E+01	-.879E+01	-.915E+01
.70	-.125E+02	-.135E+02	-.157E+02	-.176E+02	-.180E+02
.80	-.286E+02	-.303E+02	-.336E+02	-.358E+02	-.359E+02
.90	-.726E+02	-.735E+02	-.745E+02	-.736E+02	-.730E+02
.94	-.110E+03	-.108E+03	-.104E+03	-.989E+02	-.977E+02
.98	-.172E+03	-.163E+03	-.146E+03	-.134E+03	-.132E+03
1.00	-.220E+03	-.204E+03	-.175E+03	-.157E+03	-.155E+03

Table 10. Comparison of shear stress and normal stress for a single lap joint subjected to bending ($M_0 \neq 0$, $Q_0 = N_0 = \Delta T = 0$) for $h_0 = 2.54\times10^{-3}$mm and 5.08×10^{-3}mm, where $h_1 = .762$mm, $h_2 = 2.286$mm, $\ell = 12.7$mm, $T = 21°C$, and $\beta = 2.54\times10^{-2}$m.

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\sigma(x,t)/(M_0\beta^2)$	$h_0 = 2.54\times10^{-2}$ mm		
-1.0	-.286E+04	-.279E+04	-.267E+04	-.262E+04	-.263E+04
-.98	-.170E+03	-.186E+03	-.213E+03	-.229E+03	-.231E+03
-.94	.157E+03	.159E+03	.163E+03	.165E+03	.166E+03
-.90	.107E+03	.109E+03	.113E+03	.116E+03	.116E+03
-.80	.429E+02	.429E+02	.430E+02	.434E+02	.436E+02
-.70	.192E+02	.190E+02	.186E+02	.183E+02	.183E+02
-.60	.884E+01	.875E+01	.854E+01	.829E+01	.823E+01
-.40	.190E+01	.190E+01	.188E+01	.184E+01	.182E+01
-.20	.407E+00	.409E+00	.412E+00	.412E+00	.410E+00
0.00	.653E-01	.655E-01	.658E-01	.652E-01	.643E-01
.20	-.907E-01	-.941E-01	-.102E+00	-.112E+00	-.116E+00
.40	-.504E+00	-.517E+00	-.547E+00	-.580E+00	-.591E+00
.60	-.235E+01	-.238E+01	-.246E+01	-.253E+01	-.254E+01
.70	-.503E+01	-.507E+01	-.515E+01	-.518E+01	-.517E+01
.80	-.106E+02	-.106E+02	-.105E+02	-.103E+02	-.102E+02
.90	-.207E+02	-.203E+02	-.195E+02	-.187E+02	-.186E+02
.94	-.238E+02	-.230E+02	-.215E+02	-.204E+02	-.202E+02
.98	.473E+02	.494E+02	.526E+02	.535E+02	.533E+02
1.00	.514E+03	.492E+03	.454E+03	.432E+03	.429E+03
		$\sigma(x,t)/(M_0\beta^2)$	$h_0 = 5.08\times10^{-2}$ mm		
-1.0	-.207E+04	-.201E+04	-.193E+04	-.190E+04	-.190E+04
-.98	-.309E+03	-.322E+03	-.343E+03	-.355E+03	-.356E+03
-.94	.148E+03	.148E+03	.147E+03	.145E+03	.145E+03
-.90	.117E+03	.119E+03	.124E+03	.127E+03	.127E+03
-.80	.445E+02	.450E+02	.460E+02	.471E+02	.474E+02
-.70	.189E+02	.187E+02	.185E+02	.184E+02	.184E+02
-.60	.856E+01	.840E+01	.806E+01	.777E+01	.773E+01
-.40	.189E+01	.186E+01	.178E+01	.167E+01	.163E+01
-.20	.424E+00	.423E+00	.416E+00	.397E+00	.385E+00
0.00	.669E-01	.660E-01	.627E-01	.550E-01	.494E-01
.20	-.115E+00	-.124E+00	-.143E+00	-.166E+00	-.175E+00
.40	-.598E+00	-.623E+00	-.680E+00	-.736E+00	-.749E+00
.60	-.261E+01	-.266E+01	-.274E+01	-.279E+01	-.279E+01
.70	-.536E+01	-.538E+01	-.539E+01	-.533E+01	-.529E+01
.80	-.107E+02	-.106E+02	-.103E+02	-.992E+01	-.983E+01
.90	-.194E+02	-.189E+02	-.179E+02	-.171E+02	-.169E+02
.94	-.182E+02	-.171E+02	-.152E+02	-.140E+02	-.138E+02
.98	.674E+02	.679E+02	.682E+02	.673E+02	.669E+02
1.00	.353E+03	.337E+03	.309E+03	.293E+03	.291E+03

Table 10. Continued

(a) $\tau(x,t)/(\Delta T/\beta)$ h_1 = 1.27 mm

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.343E+02	.314E+02	.265E+02	.234E+02	.231E+02
-.98	.285E+02	.266E+02	.230E+02	.207E+02	.204E+02
-.94	.198E+02	.190E+02	.175E+02	.162E+02	.160E+02
-.90	.137E+02	.136E+02	.132E+02	.127E+02	.125E+02
-.80	.562E+01	.599E+01	.660E+01	.686E+01	.683E+01
-.70	.235E+01	.267E+01	.328E+01	.371E+01	.375E+01
-.60	.101E+01	.121E+01	.163E+01	.200E+01	.207E+01
-.40	.206E+00	.265E+00	.409E+00	.583E+00	.634E+00
-.20	.451E-01	.594E-01	.989E-01	.158E+00	.181E+00
0.00	.998E-12	.699E-12	.456E-12	.366E-12	.510E-12
.20	-.451E-01	-.594E-01	-.989E-01	-.158E+00	-.181E+00
.40	-.206E+00	-.265E+00	-.409E+00	-.583E+00	-.634E+00
.60	-.101E+01	-.121E+01	-.163E+01	-.200E+01	-.207E+01
.70	-.235E+01	-.267E+01	-.328E+01	-.371E+01	-.375E+01
.80	-.562E+01	-.599E+01	-.660E+01	-.686E+01	-.683E+01
.90	-.137E+02	-.136E+02	-.132E+02	-.127E+02	-.125E+02
.94	-.198E+02	-.190E+02	-.175E+02	-.162E+02	-.160E+02
.98	-.285E+02	-.266E+02	-.230E+02	-.207E+02	-.204E+02
1.00	-.343E+02	-.314E+02	-.265E+02	-.234E+02	-.231E+02

(b) $\tau(x,t)/(\Delta T/\beta)$ h_1 = 2.286 mm

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	.391E+02	.359E+02	.302E+02	.267E+02	.263E+02
-.98	.333E+02	.309E+02	.267E+02	.240E+02	.236E+02
-.94	.241E+02	.231E+02	.210E+02	.193E+02	.190E+02
-.90	.175E+02	.172E+02	.164E+02	.156E+02	.154E+02
-.80	.798E+01	.836E+01	.894E+01	.910E+01	.904E+01
-.70	.371E+01	.411E+01	.486E+01	.532E+01	.534E+01
-.60	.176E+01	.205E+01	.264E+01	.311E+01	.317E+01
-.40	.433E+00	.540E+00	.788E+00	.106E+01	.112E+01
-.20	.111E+00	.142E+00	.221E+00	.326E+00	.360E+00
0.00	-.525E-12	-.174E-11	.665E-13	.888E-12	-.724E-12
.20	-.111E+00	-.142E+00	-.221E+00	-.326E+00	-.360E+00
.40	-.433E+00	-.540E+00	-.788E+00	-.106E+01	-.112E+01
.60	-.176E+01	-.205E+01	-.264E+01	-.311E+01	-.317E+01
.70	-.371E+01	-.411E+01	-.486E+01	-.532E+01	-.534E+01
.80	-.798E+01	-.836E+01	-.894E+01	-.910E+01	-.904E+01
.90	-.175E+02	-.172E+02	-.164E+02	-.156E+02	-.154E+02
.94	-.241E+02	-.231E+02	-.210E+02	-.193E+02	-.190E+02
.98	-.333E+02	-.309E+02	-.267E+02	-.240E+02	-.236E+02
1.00	-.391E+02	-.359E+02	-.302E+02	-.267E+02	-.263E+02

Table 11. Comparison of shear stress and normal stress resulting from a temperature increase ($\Delta T \neq 0$, $M_0=N_0=Q_0=0$) in a cover plate for h_1=1.27mm and 2.286mm, where h_2= 2.286mm, ℓ=12.7mm, h_0=.1016mm, T=21°C, and β= $(2.54\times10^{-2}\text{m})(5/9°\text{C})/(4.448\text{N})$.

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.

(c) $\sigma(x,t)/(\Delta T/\beta)$ h_1 = 1.27 mm

-1.0	.115E+02	.108E+02	.956E+01	.880E+01	.870E+01
-.98	.498E+01	.481E+01	.448E+01	.422E+01	.417E+01
-.94	.458E+00	.524E+00	.625E+00	.659E+00	.655E+00
-.90	-.557E+00	-.488E+00	-.364E+00	-.287E+00	-.280E+00
-.80	-.719E+00	-.686E+00	-.619E+00	-.562E+00	-.551E+00
-.70	-.509E+00	-.500E+00	-.475E+00	-.447E+00	-.439E+00
-.60	-.324E+00	-.326E+00	-.324E+00	-.316E+00	-.312E+00
-.40	-.116E+00	-.121E+00	-.131E+00	-.137E+00	-.138E+00
-.20	-.416E-01	-.446E-01	-.514E-01	-.585E-01	-.603E-01
0.00	-.241E-01	-.262E-01	-.314E-01	-.377E-01	-.396E-01
.20	-.416E-01	-.446E-01	-.514E-01	-.585E-01	-.603E-01
.40	-.116E+00	-.121E+00	-.131E+00	-.137E+00	-.138E+00
.60	-.324E+00	-.326E+00	-.324E+00	-.316E+00	-.312E+00
.70	-.509E+00	-.500E+00	-.475E+00	-.447E+00	-.439E+00
.80	-.719E+00	-.686E+00	-.619E+00	-.562E+00	-.551E+00
.90	-.557E+00	-.488E+00	-.364E+00	-.287E+00	-.280E+00
.94	.458E+00	.524E+00	.625E+00	.659E+00	.655E+00
.98	.498E+01	.481E+01	.448E+01	.422E+01	.417E+01
1.00	.115E+02	.108E+02	.956E+01	.880E+01	.870E+01

(d) $\sigma(x,t)/(\Delta T/\beta)$ h_1 = 2.286 mm

-1.0	-.133E+02	-.125E+02	-.112E+02	-.103E+02	-.102E+02
-.98	-.703E+01	-.676E+01	-.625E+01	-.587E+01	-.580E+01
-.94	-.151E+01	-.156E+01	-.162E+01	-.161E+01	-.160E+01
-.90	.248E+00	.158E+00	.538E-02	-.750E-01	-.789E-01
-.80	.936E+00	.883E+00	.780E+00	.698E+00	.685E+00
-.70	.776E+00	.754E+00	.703E+00	.653E+00	.642E+00
-.60	.563E+00	.558E+00	.542E+00	.519E+00	.511E+00
-.40	.262E+00	.268E+00	.279E+00	.284E+00	.283E+00
-.20	.125E+00	.131E+00	.144E+00	.155E+00	.157E+00
0.00	.873E-01	.926E-01	.104E+00	.117E+00	.120E+00
.20	.125E+00	.131E+00	.144E+00	.155E+00	.157E+00
.40	.262E+00	.268E+00	.279E+00	.284E+00	.283E+00
.60	.563E+00	.558E+00	.542E+00	.519E+00	.511E+00
.70	.776E+00	.754E+00	.703E+00	.653E+00	.642E+00
.80	.936E+00	.883E+00	.780E+00	.698E+00	.685E+00
.90	.248E+00	.158E+00	.538E-02	-.750E-01	-.789E-01
.94	-.151E+01	-.156E+01	-.162E+01	-.161E+01	-.160E+01
.98	-.703E+01	-.676E+01	-.625E+01	-.587E+01	-.580E+01
1.00	-.133E+02	-.125E+02	-.112E+02	-.103E+02	-.102E+02

Table 11. Continued

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\tau(x,t)/(M_o/\beta^2)$	REISSNER THEORY		
-1.0	.488E+03	.438E+03	.351E+03	.301E+03	.296E+03
-.98	.360E+03	.330E+03	.276E+03	.240E+03	.236E+03
-.94	.181E+03	.175E+03	.161E+03	.147E+03	.145E+03
-.90	.796E+02	.835E+02	.878E+02	.864E+02	.851E+02
-.80	-.901E+01	-.237E+01	.965E+01	.169E+02	.174E+02
-.70	-.182E+02	-.147E+02	-.755E+01	-.165E+01	-.780E+00
-.60	-.130E+02	-.116E+02	-.836E+01	-.491E+01	-.414E+01
-.40	-.416E+01	-.408E+01	-.371E+01	-.293E+01	-.258E+01
-.20	-.117E+01	-.123E+01	-.130E+01	-.124E+01	-.115E+01
0.00	-.441E+00	-.504E+00	-.651E+00	-.810E+00	-.847E+00
.20	-.633E+00	-.758E+00	-.107E+01	-.147E+01	-.161E+01
.40	-.218E+01	-.252E+01	-.332E+01	-.423E+01	-.448E+01
.60	-.848E+01	-.937E+01	-.112E+02	-.129E+02	-.132E+02
.70	-.170E+02	-.182E+02	-.207E+02	-.224E+02	-.226E+02
.80	-.344E+02	-.357E+02	-.379E+02	-.387E+02	-.385E+02
.90	-.717E+02	-.710E+02	-.689E+02	-.661E+02	-.654E+02
.94	-.972E+02	-.940E+02	-.874E+02	-.817E+02	-.806E+02
.98	-.134E+03	-.126E+03	-.111E+03	-.101E+03	-.996E+02
1.00	-.159E+03	-.147E+03	-.126E+03	-.113E+03	-.111E+03
		$\tau(x,t)/(M_o/\beta^2)$	CLASSICAL THEORY		
-1.0	.428E+03	.382E+03	.303E+03	.258E+03	.254E+03
-.96	.299E+03	.273E+03	.227E+03	.198E+03	.194E+03
-.94	.121E+03	.119E+03	.114E+03	.106E+03	.104E+03
-.90	.362E+02	.426E+02	.520E+02	.545E+02	.538E+02
-.80	.483E+01	.844E+01	.154E+02	.202E+02	.205E+02
-.70	.778E+01	.914E+01	.121E+02	.148E+02	.152E+02
-.60	.370E+01	.459E+01	.657E+01	.855E+01	.897E+01
-.40	.441E+00	.673E+00	.129E+01	.214E+01	.242E+01
-.20	.721E-01	.120E+00	.267E+00	.520E+00	.637E+00
0.00	-.785E-02	-.134E-01	-.299E-01	-.565E-01	-.663E-01
.20	-.122E+00	-.196E+00	-.411E+00	-.746E+00	-.884E+00
.40	-.795E+00	-.113E+01	-.195E+01	-.295E+01	-.324E+01
.60	-.499E+01	-.619E+01	-.868E+01	-.108E+02	-.112E+02
.70	-.130E+02	-.149E+02	-.184E+02	-.207E+02	-.209E+02
.80	-.342E+02	-.360E+02	-.387E+02	-.397E+02	-.395E+02
.90	-.800E+02	-.785E+02	-.750E+02	-.713E+02	-.704E+02
.94	-.107E+03	-.103E+03	-.943E+02	-.875E+02	-.863E+02
.98	-.143E+03	-.134E+03	-.118E+03	-.107E+03	-.105E+03
1.00	-.168E+03	-.155E+03	-.133E+03	-.118E+03	-.117E+03

Table 12. Comparison of shear stress and normal stress for a single lap joint subjected to bending ($M_o \neq 0$), $N_o=Q_o=\Delta T=0$) for Reissner and for classical plate theories, where $h_1=$ .762mm, h_2=2.286mm, h_o=.1016mm, ℓ=12.7mm, T=21°C, and $\beta=2.54\times10^{-2}$m.

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.

$\sigma(x,t)/(M_0/\beta^2)$ REISSNER THEORY

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.149E+04	-.145E+04	-.140E+04	-.137E+04	-.137E+04
-.98	-.386E+03	-.393E+03	-.405E+03	-.411E+03	-.412E+03
-.94	.111E+03	.107E+03	.998E+02	.954E+02	.950E+02
-.90	.123E+03	.124E+03	.127E+03	.128E+03	.128E+03
-.80	.488E+02	.499E+02	.520E+02	.537E+02	.540E+02
-.70	.191E+02	.191E+02	.193E+02	.197E+02	.198E+02
-.60	.813E+01	.796E+01	.766E+01	.750E+01	.752E+01
-.40	.176E+01	.168E+01	.150E+01	.132E+01	.129E+01
-.20	.417E+00	.400E+00	.353E+00	.284E+00	.260E+00
0.00	.573E-01	.496E-01	.272E-01	-.839E-02	-.241E-01
.20	-.173E+00	-.192E+00	-.235E+00	-.282E+00	-.295E+00
.40	-.767E+00	-.807E+00	-.888E+00	-.950E+00	-.956E+00
.60	-.294E+01	-.297E+01	-.300E+01	-.297E+01	-.294E+01
.70	-.566E+01	-.561E+01	-.548E+01	-.528E+01	-.522E+01
.80	-.107E+02	-.104E+02	-.994E+01	-.948E+01	-.938E+01
.90	-.172E+02	-.165E+02	-.153E+02	-.144E+02	-.143E+02
.94	-.918E+01	-.800E+01	-.601E+01	-.498E+01	-.493E+01
.98	.747E+02	.737E+02	.715E+02	.693E+02	.688E+02
1.00	.243E+03	.231E+03	.211E+03	.199E+03	.198E+03

$\sigma(x,t)/(M_0/\beta^2)$ CLASSICAL THEORY

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
-1.0	-.156E+04	-.152E+04	-.146E+04	-.147E+04	-.143E+04
-.98	-.739E+03	-.732E+03	-.720E+03	-.715E+03	-.715E+03
-.94	.122E+03	.109E+03	.869E+02	.754E+02	.745E+02
-.90	.322E+03	.315E+03	.303E+03	.296E+03	.296E+03
-.80	.773E+02	.821E+02	.906E+02	.957E+02	.962E+02
-.70	-.147E+02	-.145E+02	-.140E+02	-.133E+02	-.131E+02
-.60	-.491E+01	-.562E+01	-.692E+01	-.776E+01	-.782E+01
-.40	.100E+00	.968E-01	.681E-01	.384E-02	-.161E-01
-.20	-.281E-01	-.415E-01	-.773E-01	-.128E+00	-.147E+00
0.00	-.674E-02	-.121E-01	-.290E-01	-.602E-01	-.757E-01
.20	-.279E-01	-.431E-01	-.839E-01	-.140E+00	-.158E+00
.40	-.213E+00	-.276E+00	-.414E+00	-.539E+00	-.559E+00
.60	-.561E+00	-.704E+00	-.960E+00	-.109E+01	-.108E+01
.70	-.247E+01	-.277E+01	-.320E+01	-.326E+01	-.319E+01
.80	-.205E+02	-.203E+02	-.196E+02	-.186E+02	-.184E+02
.90	-.391E+02	-.360E+02	-.309E+02	-.282E+02	-.280E+02
.94	.314E+01	.496E+01	.757E+01	.833E+01	.822E+01
.98	.122E+03	.117E+03	.108E+03	.102E+03	.101E+03
1.00	.218E+03	.206E+03	.187E+03	.176E+03	.174E+03

Table 12. Continued

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\tau(x,t)/(\Delta T/\beta)$	REISSNER THEORY		
-1.0	.323E+02	.297E+02	.252E+02	.224E+02	.221E+02
-.98	.266E+02	.249E+02	.218E+02	.197E+02	.195E+02
-.94	.184E+02	.178E+02	.165E+02	.154E+02	.152E+02
-.90	.130E+02	.129E+02	.126E+02	.121E+02	.120E+02
-.80	.571E+01	.600E+01	.650E+01	.672E+01	.669E+01
-.70	.266E+01	.291E+01	.340E+01	.376E+01	.379E+01
-.60	.128E+01	.145E+01	.180E+01	.211E+01	.217E+01
-.40	.316E+00	.374E+00	.510E+00	.666E+00	.712E+00
-.20	.756E-01	.926E-01	.136E+00	.193E+00	.214E+00
0.00	.927E-12	.928E-12	.194E-11	.782E-12	.917E-12
.20	-.756E-01	-.926E-01	-.136E+00	-.193E+00	-.214E+00
.40	-.316E+00	-.374E+00	-.510E+00	-.666E+00	-.712E+00
.60	-.128E+01	-.145E+01	-.180E+01	-.211E+01	-.217E+01
.70	-.266E+01	-.291E+01	-.340E+01	-.376E+01	-.379E+01
.80	-.571E+01	-.600E+01	-.650E+01	-.672E+01	-.669E+01
.90	-.130E+02	-.129E+02	-.126E+02	-.121E+02	-.120E+02
.94	-.184E+02	-.178E+02	-.165E+02	-.154E+02	-.152E+02
.98	-.266E+02	-.249E+02	-.218E+02	-.197E+02	-.195E+02
1.00	-.323E+02	-.297E+02	-.252E+02	-.224E+02	-.221E+02
		$\tau(x,t)/(\Delta T/\beta)$	CLASSICAL THEORY		
-1.0	.331E+02	.305E+02	.258E+02	.229E+02	.226E+02
-.98	.275E+02	.257E+02	.224E+02	.202E+02	.199E+02
-.94	.194E+02	.187E+02	.172E+02	.159E+02	.157E+02
-.90	.139E+02	.137E+02	.132E+02	.126E+02	.125E+02
-.80	.583E+01	.616E+01	.670E+01	.692E+01	.689E+01
-.70	.228E+01	.261E+01	.324E+01	.366E+01	.370E+01
-.60	.884E+00	.110E+01	.154E+01	.192E+01	.198E+01
-.40	.139E+00	.198E+00	.343E+00	.521E+00	.573E+00
-.20	.211E-01	.341E-01	.713E-01	.129E+00	.153E+00
0.00	-.195E-12	-.258E-12	-.299E-12	-.173E-12	-.180E-12
.20	-.211E-01	-.341E-01	-.713E-01	-.129E+00	-.153E+00
.40	-.139E+00	-.198E+00	-.343E+00	-.521E+00	-.573E+00
.60	-.884E+00	-.110E+01	-.154E+01	-.192E+01	-.198E+01
.70	-.228E+01	-.261E+01	-.324E+01	-.366E+01	-.370E+01
.80	-.583E+01	-.616E+01	-.670E+01	-.692E+01	-.689E+01
.90	-.139E+02	-.137E+02	-.132E+02	-.126E+02	-.125E+02
.94	-.194E+02	-.187E+02	-.172E+02	-.159E+02	-.157E+02
.98	-.275E+02	-.257E+02	-.224E+02	-.202E+02	-.199E+02
1.00	-.331E+02	-.305E+02	-.258E+02	-.229E+02	-.226E+02

Table 13. Comparison of shear stress and normal stress resulting from a temperature increase ($\Delta T \neq 0$, $M_0=N_0=Q_0=0$) in a cover plate for Reissner and for Classical plate theories, where $h_1=.762$mm, $h_2=2.286$mm, $h_0=.1016$mm, $\ell=12.7$mm, T=21°C, and $\beta=(2.54 \times 10^{-2}\text{m})(5/9°\text{C})/(4.448\text{N})$.

x/ℓ	t=0	t=5 min.	t=20 min.	t=1 hr.	t=3 hr.
		$\sigma(x,t)/(\Delta T/\beta)$	REISSNER THEORY		
-1.0	.242E+02	.226E+02	.199E+02	.181E+02	.179E+02
-.98	.825E+01	.800E+01	.749E+01	.704E+01	.696E+01
-.94	-.301E+00	-.145E+00	.111E+00	.236E+00	.239E+00
-.90	-.152E+01	-.139E+01	-.115E+01	-.986E+00	-.966E+00
-.80	-.125E+01	-.120E+01	-.109E+01	-.993E+00	-.974E+00
-.70	-.739E+00	-.731E+00	-.706E+00	-.669E+00	-.658E+00
-.60	-.405E+00	-.412E+00	-.421E+00	-.417E+00	-.413E+00
-.40	-.111E+00	-.119E+00	-.135E+00	-.148E+00	-.150E+00
-.20	-.310E-01	-.350E-01	-.440E-01	-.539E-01	-.566E-01
0.00	-.151E-01	-.177E-01	-.239E-01	-.317E-01	-.344E-01
.20	-.310E-01	-.350E-01	-.440E-01	-.539E-01	-.566E-01
.40	-.111E+00	-.119E+00	-.135E+00	-.148E+00	-.150E+00
.60	-.405E+00	-.412E+00	-.421E+00	-.417E+00	-.413E+00
.70	-.739E+00	-.731E+00	-.706E+00	-.669E+00	-.658E+00
.80	-.125E+01	-.120E+01	-.109E+01	-.993E+00	-.974E+00
.90	-.152E+01	-.139E+01	-.115E+01	-.986E+00	-.966E+00
.94	-.301E+00	-.145E+00	.111E+00	.236E+00	.239E+00
.98	.825E+01	.800E+01	.749E+01	.704E+01	.696E+01
1.00	.242E+02	.226E+02	.199E+02	.181E+02	.179E+02
		$\sigma(x,t)/(\Delta T/\beta)$	CLASSICAL THEORY		
-1.0	.191E+02	.176E+02	.151E+02	.135E+02	.133E+02
-.98	.123E+02	.115E+02	.101E+02	.918E+01	.905E+01
-.94	.191E+01	.206E+01	.226E+01	.226E+01	.223E+01
-.90	-.296E+01	-.253E+01	-.180E+01	-.141E+01	-.139E+01
-.80	-.259E+01	-.250E+01	-.228E+01	-.206E+01	-.202E+01
-.70	-.582E+00	-.631E+00	-.698E+00	-.699E+00	-.685E+00
-.60	-.150E+00	-.182E+00	-.240E+00	-.271E+00	-.270E+00
-.40	-.349E-01	-.455E-01	-.690E-01	-.910E-01	-.945E-01
-.20	-.506E-02	-.782E-02	-.152E-01	-.254E-01	-.288E-01
0.00	-.158E-02	-.274E-02	-.629E-02	-.125E-01	-.155E-01
.20	-.506E-02	-.782E-02	-.152E-01	-.254E-01	-.288E-01
.40	-.349E-01	-.455E-01	-.690E-01	-.910E-01	-.945E-01
.60	-.150E+00	-.182E+00	-.240E+00	-.271E+00	-.270E+00
.70	-.582E+00	-.631E+00	-.698E+00	-.699E+00	-.685E+00
.80	-.259E+01	-.250E+01	-.228E+01	-.206E+01	-.202E+01
.90	-.296E+01	-.253E+01	-.180E+01	-.141E+01	-.139E+01
.94	.191E+01	.206E+01	.226E+01	.226E+01	.223E+01
.98	.123E+02	.115E+02	.101E+02	.918E+01	.905E+01
1.00	.191E+02	.176E+02	.151E+02	.135E+02	.133E+02

Table 13. Continued

Time in Seconds	Data From Fig. (14) 1 Unit=.01245 cm	Calculated Value Using Eqn. (122)
0.00	.630000E+01	.630000E+01
1.00	.650000E+01	.650766E+01
3.00	.660000E+01	.658218E+01
5.00	.663000E+01	.663721E+01
10.00	.672000E+01	.672743E+01
15.00	.678000E+01	.678505E+01
20.00	.683000E+01	.682846E+01
25.00	.687000E+01	.686415E+01
30.00	.690000E+01	.689468E+01
35.00	.692500E+01	.692126E+01
40.00	.695000E+01	.694460E+01
45.00	.696500E+01	.696520E+01
50.00	.698000E+01	.698346E+01
55.00	.700000E+01	.699971E+01
60.00	.701500E+01	.701422E+01
65.00	.702500E+01	.702725E+01
70.00	.703500E+01	.703899E+01
75.00	.704500E+01	.704963E+01
80.00	.705500E+01	.705930E+01
85.00	.706500E+01	.706816E+01
90.00	.707500E+01	.707629E+01
95.00	.708500E+01	.708381E+01
100.00	.709500E+01	.709080E+01
150.00	.713000E+01	.714284E+01
200.00	.718500E+01	.718049E+01
250.00	.723000E+01	.721290E+01
300.00	.725000E+01	.724222E+01
350.00	.727000E+01	.726910E+01
400.00	.729000E+01	.729380E+01
450.00	.732000E+01	.731653E+01
500.00	.733000E+01	.733744E+01
550.00	.735500E+01	.735668E+01
600.00	.737000E+01	.737438E+01
650.00	.739000E+01	.739067E+01
700.00	.741000E+01	.740566E+01
800.00	.743000E+01	.743214E+01
900.00	.745000E+01	.745455E+01
1000.00	.747000E+01	.747353E+01
1100.00	.749000E+01	.748960E+01
1200.00	.750500E+01	.750320E+01
1300.00	.752000E+01	.751472E+01

THE SUM OF THE SQUARES IS .145251E-02
THE MAXIMUM DIFFERENCE IS .178230E=01 (T=3 sec.)

TABLE 15. DATA FIT OF CREEP CURVE

Time In Seconds	T(0,t) for Generation Per Cycle °C	T(0,t) for Generation Per Unit Time (No coupling) °C	T(0,t) for Generation Per Unit Time (With Coupling) °C
0	22.000	22.000	22.000
100	22.940	22.951	22.873
200	23.804	23.816	23.738
300	24.536	24.546	24.474
400	25.141	25.151	25.086
500	25.639	25.648	25.590
600	26.050	26.057	26.005
700	26.387	26.393	26.347
800	26.664	26.670	26.629
900	26.892	26.897	26.861
1000	27.080	27.084	27.052
1100	27.234	27.237	27.210
1200	27.361	27.364	27.340
1300	27.466	27.468	27.447
1400	27.551	27.553	27.536
1500	27.622	27.623	27.609
1600	27.680	27.681	27.668
1700	27.728	27.729	27.718
1800	27.767	27.768	27.759
1900	27.799	27.800	27.792
2000	27.826	27.826	27.820
2100	27.848	27.848	27.843
2200	27.866	27.866	27.862
2300	27.880	27.881	27.877
2400	27.893	27.893	27.890
2500	27.903	27.903	27.901
2600	27.911	27.911	27.910
2700	27.918	27.918	27.917
2800	27.923	27.923	27.923
2900	27.928	27.928	27.928
3000	27.931	27.932	27.932
3100	27.935	27.935	27.935
3200	27.937	27.937	27.938
3300	27.939	27.939	27.940
3400	27.941	27.941	27.942
3500	27.942	27.942	27.944
3600	27.944	27.943	27.945
3700	27.944	27.944	27.946
3800	27.945	27.945	27.947
3900	27.946	27.946	27.947
4000	27.946	27.946	27.948

TABLE 16 - COMPARISON OF TEMPERATURE PROFILE FOR THREE DIFFERENT SOLUTIONS OF THE ENERGY EQUATION. CYCLING FREQUENCY IS 10 HERTZ.

Time In Seconds	T(0,t) for Generation per Cycle °C	T(0,t) for Generation per Unit Time (No Coupling) °C	T(0,t) for Generation per Unit Time (With Coupling) °C
0	22.000	22.000	22.000
100	30.871	30.884	30.808
200	39.044	39.057	38.981
300	45.963	45.976	45.906
400	51.689	51.700	51.637
500	56.404	56.414	56.357
600	60.283	60.291	60.241
700	63.473	63.481	63.437
800	66.098	66.104	66.065
900	68.256	68.261	68.228
1000	70.031	70.036	70.007
1100	71.492	71.495	71.470
1200	72.693	72.696	72.674
1300	73.680	73.683	73.664
1400	74.493	74.495	74.479
1500	75.161	75.163	75.150
1600	75.711	75.712	75.701
1700	76.163	76.164	76.155
1800	76.535	76.535	76.528
1900	76.840	76.841	76.835
2000	77.092	77.092	77.088
2100	77.299	77.299	77.296
2200	77.469	77.469	77.467
2300	77.609	77.609	77.608
2400	77.724	77.724	77.723
2500	77.819	77.819	77.819
2600	77.896	77.897	77.897
2700	77.961	77.961	77.962
2800	78.013	78.013	78.015
2900	78.057	78.057	78.058
3000	78.092	78.092	78.094
3100	78.121	78.121	78.124
3200	78.146	78.146	78.148
3300	78.165	78.165	78.168
3400	78.182	78.182	78.185
3500	78.195	78.195	78.198
3600	78.206	78.206	78.209
3700	78.215	78.215	78.218
3800	78.223	78.223	78.226
3900	78.229	78.229	78.232
4000	78.234	78.234	78.237

TABLE 17 - COMPARISON OF TEMPERATURE PROFILE FOR THREE DIFFERENT SOLUTIONS OF THE ENERGY EQUATION. CYCLING FREQUENCY IS 50 HERTZ.

10 HERTZ

Time In Seconds	T(0,t) for Heat Gen. Per Cycle (°C)	T(0,t) for Heat Gen. Per Unit Time (No Coupling)	T(0,t) for Heat Gen. Per Unit Time (Coupling)
0.0000	22.000	22.000	22.000
.0100	22.000	22.003	21.795
.0200	22.000	22.003	21.673
.0300	22.000	22.003	21.671
.0400	22.000	22.003	21.791
.0500	22.000	22.003	21.985
.0600	22.001	22.004	22.180
.0700	22.001	22.004	22.301
.0800	22.001	22.004	22.302
.0900	22.001	22.004	22.183
.1000	22.001	22.004	21.989

50 HERTZ

0.0000	22.000	22.000	22.000
.0020	22.000	22.002	21.805
.0040	22.000	22.003	21.683
.0060	22.001	22.004	21.679
.0080	22.001	22.004	21.795
.0100	22.001	22.004	21.985
.0120	22.001	22.004	22.177
.0140	22.001	22.005	22.297
.0160	22.001	22.005	22.300
.0180	22.002	22.005	22.183
.0200	22.002	22.005	21.992

TABLE 18. COMPARISON OF THREE SOLUTIONS OF THE ENERGY EQUATION FOR TIMES DURING THE FIRST CYCLE.

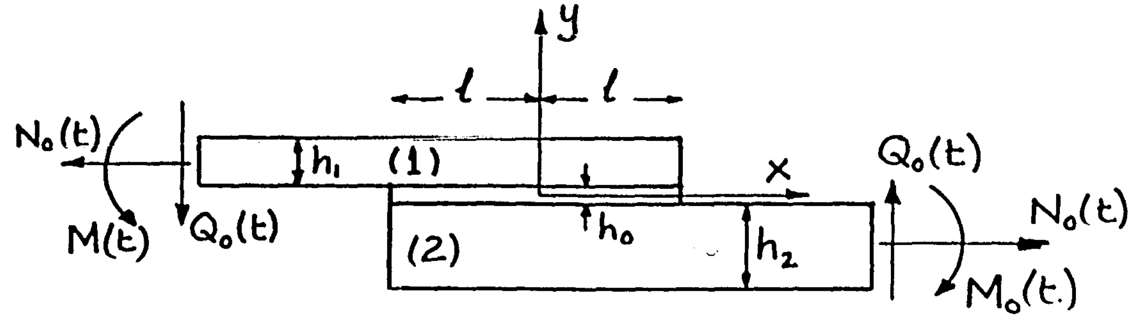
10 HERTZ

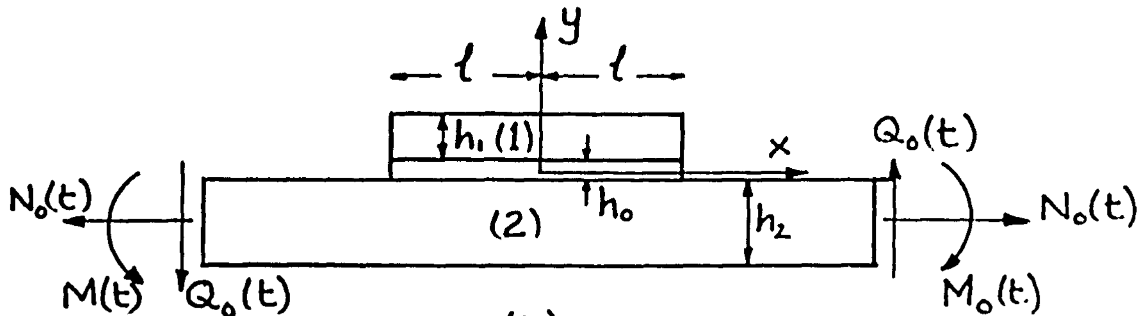
Time in Seconds	T(0,t) for Heat Gen. Per Cycle (°C)	T(0,t) for Heat Gen. Per Unit Time (No Coupling)	T(0,t) for Heat Gen. Per Unit Time (Coupling)
10.0000	22.095	22.103	22.061
10.0100	22.095	22.103	21.866
10.0200	22.095	22.103	21.745
10.0300	22.095	22.103	21.744
10.0400	22.095	22.103	21.863
10.0500	22.096	22.103	22.057
10.0600	22.096	22.103	22.252
10.0700	22.096	22.103	22.373
10.0800	22.096	22.103	22.375
10.0900	22.096	22.103	22.255
10.1000	22.096	22.103	22.062

50 HERTZ

2.0000	22.178	22.184	22.162
2.0020	22.178	22.184	21.970
2.0040	22.179	22.184	21.850
2.0060	22.179	22.184	21.848
2.0080	22.179	22.184	21.964
2.0100	22.179	22.184	22.155
2.0120	22.179	22.185	22.348
2.0140	22.180	22.185	22.468
2.0160	22.180	22.185	22.471
2.0180	22.180	22.185	22.355
2.0200	22.180	22.185	22.164

TABLE 19. COMPARISON OF THREE SOLUTIONS OF THE ENERGY EQUATION FOR TIMES DURING THE ONE HUNDREDTH CYCLE.





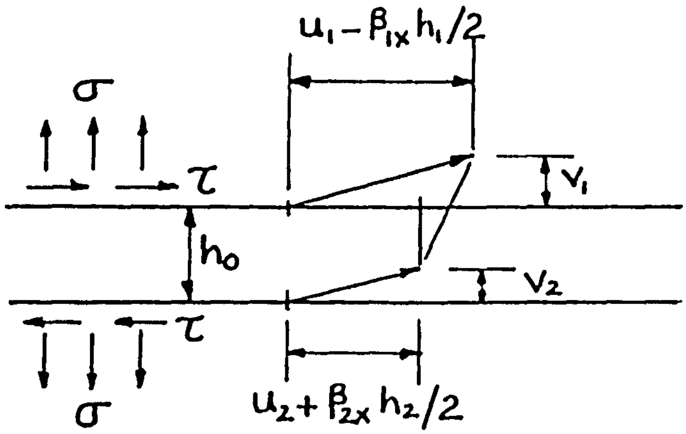


Figure 1. The geometry of the bonded joint. Figure (a) shows the single lap joint, figure (b) the cover plate, and figure (c) the kinematics of the adhesive layer.

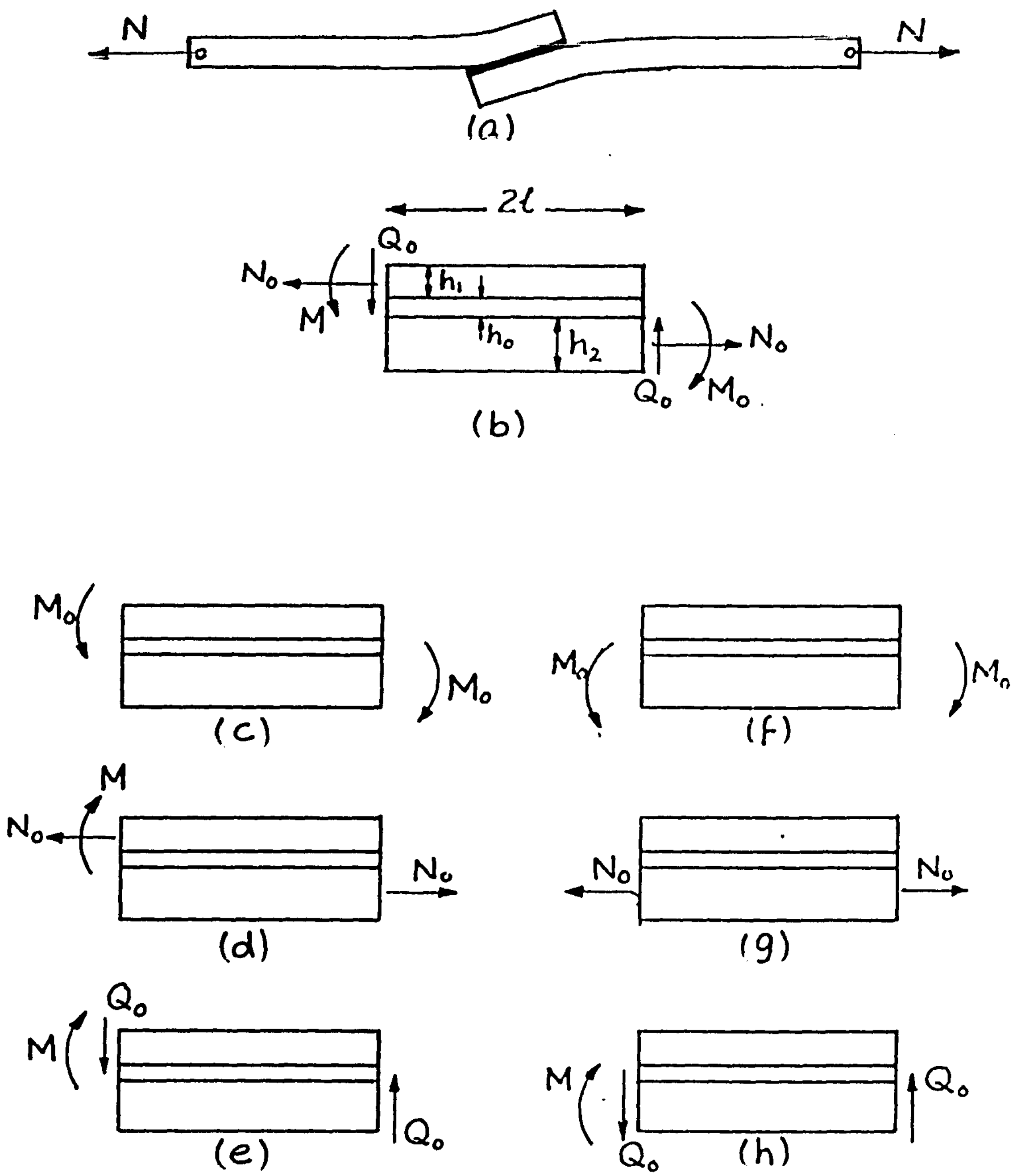


Figure 2. The effect of eccentricity of the load path (a) and the general loading in a plate theory (b) for a single lap joint. Figures c-h show the specific loadings used for the results.

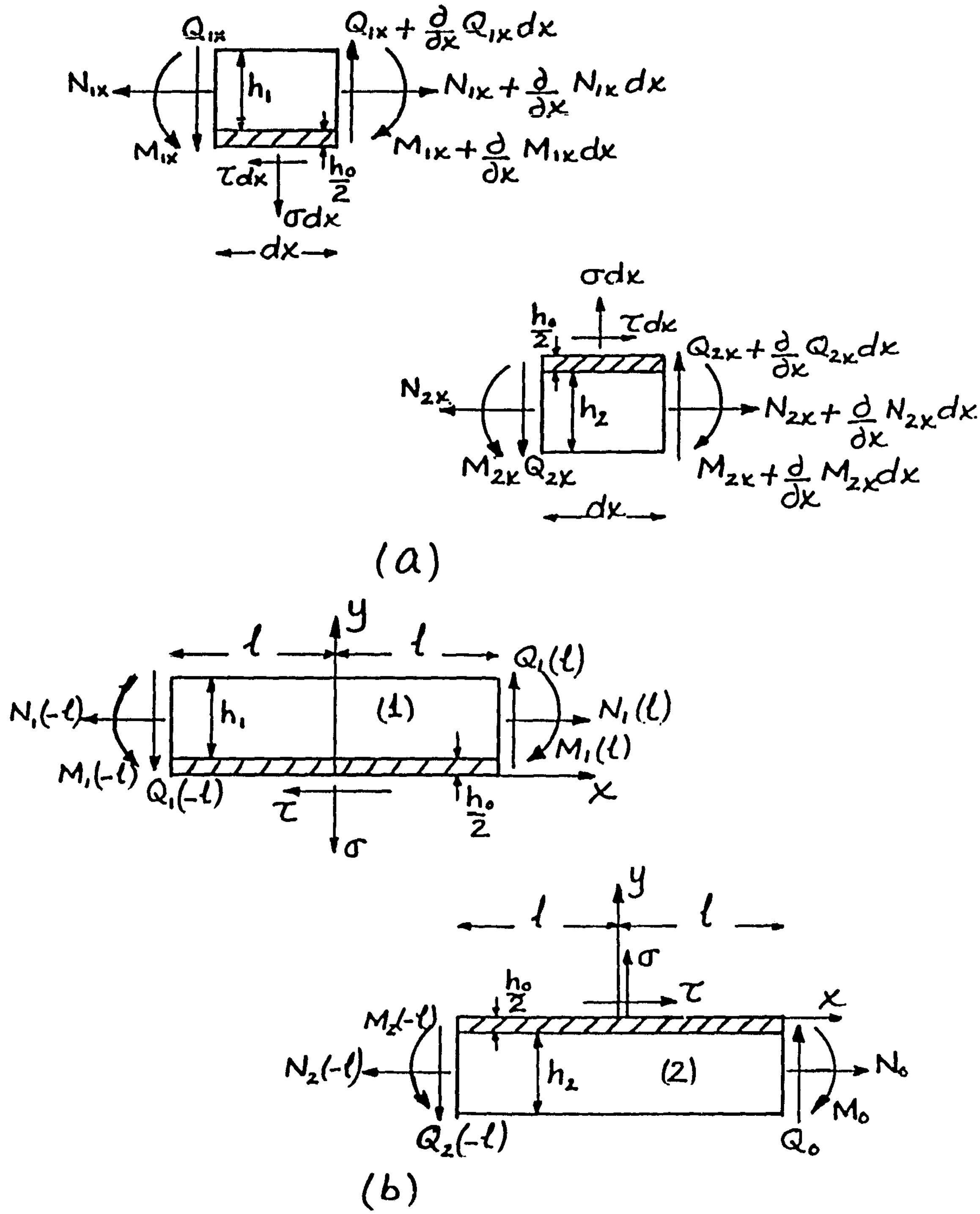


Figure 3. In figure (a) the elements used for the equilibrium equations are shown. Figure (b) shows the elements used for relations (51-53) that replace the boundary conditions.

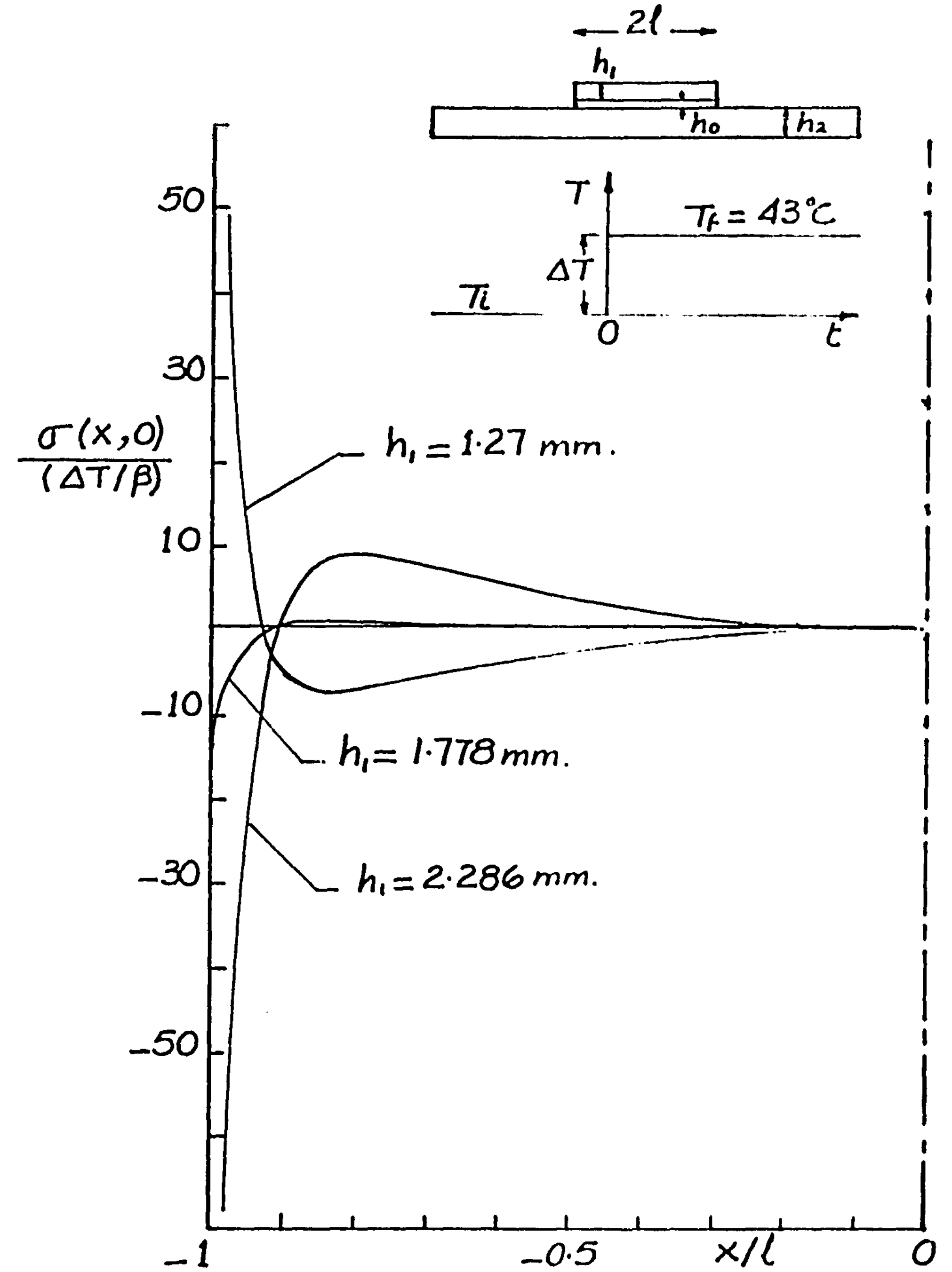


Figure 4. Distribution of the normal stress resulting from a temperature increase in a cover plate for varying values of upper plate thickness h_1. The other parameters are: h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and β= $(2.54\times10^{-2}\text{m})^2(5/9°\text{C})/(4.448\text{N})$.

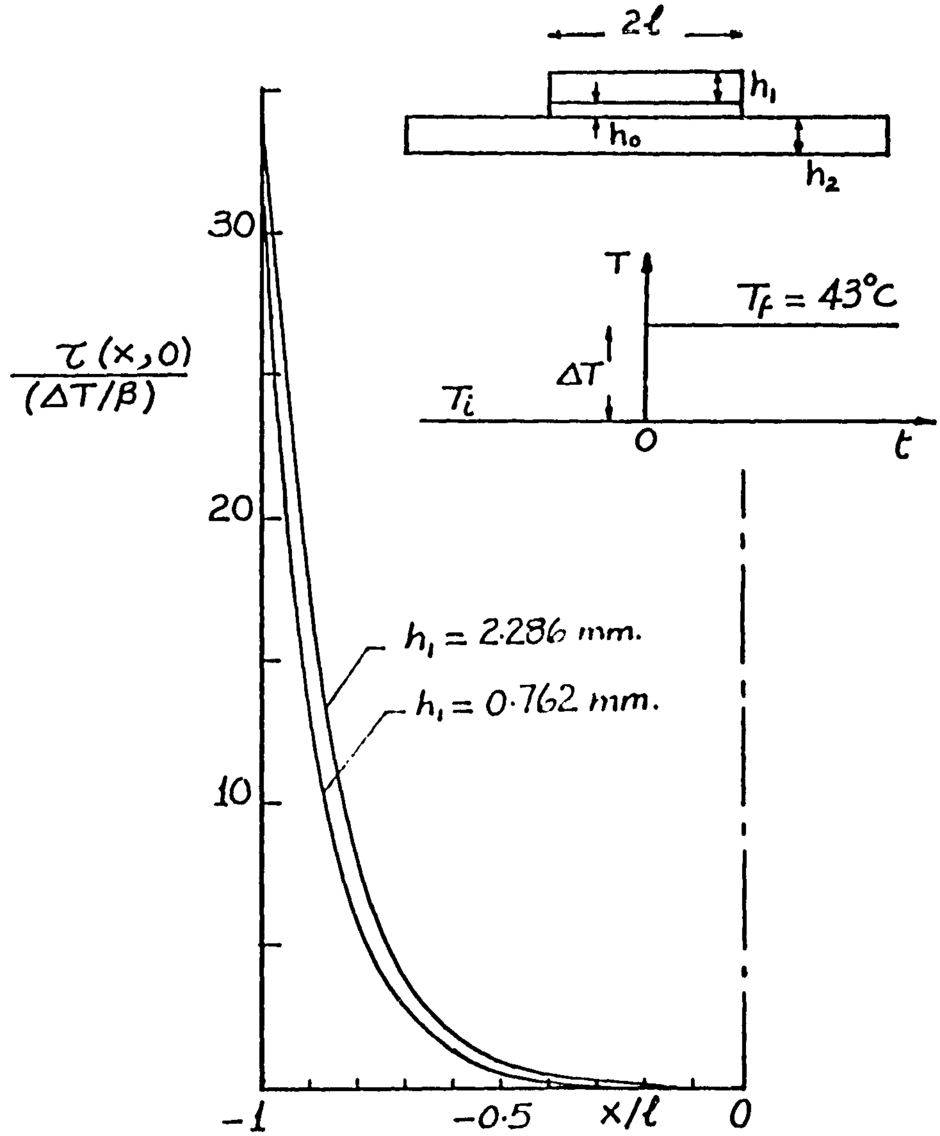


Figure 5. Distributions of the shear stress resulting from a temperature increase in a cover plate for varying values of upper plate thickness h_1. The other parameters are: h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and β=$(2.54\times10^{-2}m)^2(5/9°C)/(4.448N)$.

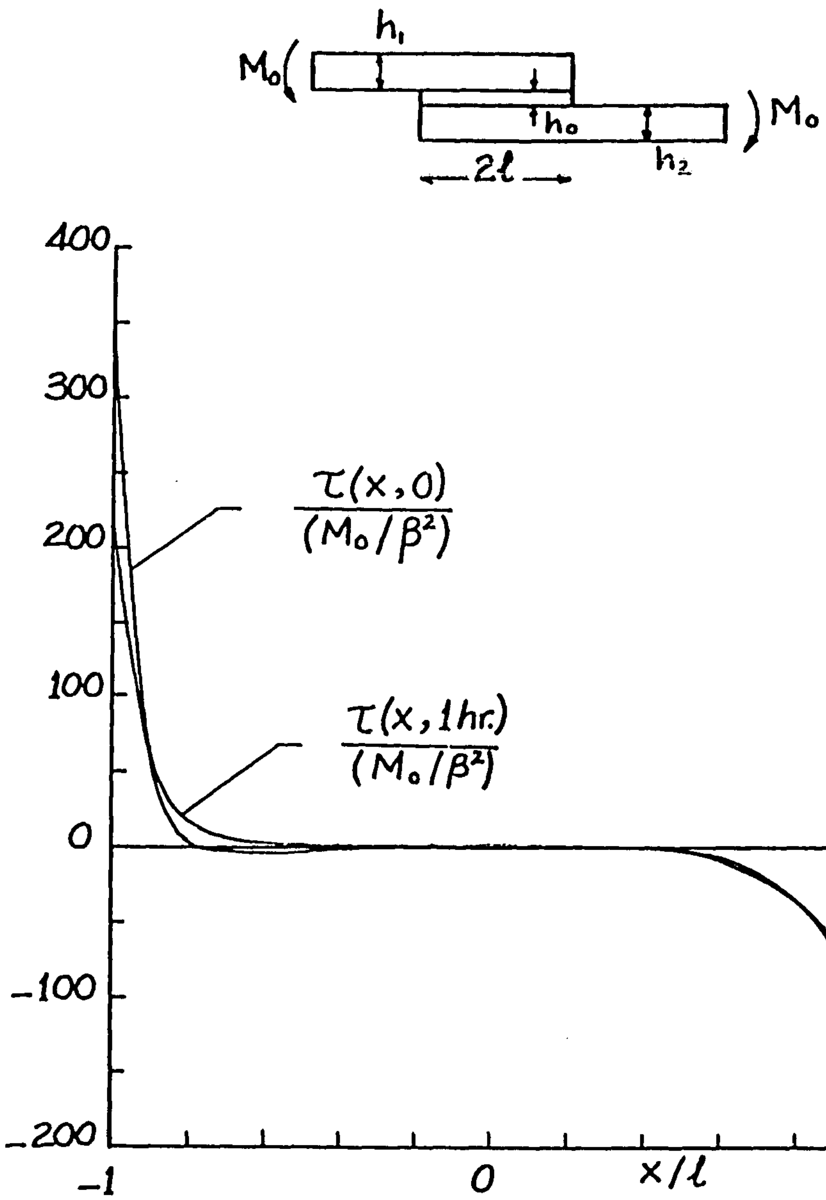


Figure 6. Distribution of shear stress in a single lap joint subjected to bending where h_1=1.27mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, T=21°C, and $\beta=2.54\times10^{-2}$m.

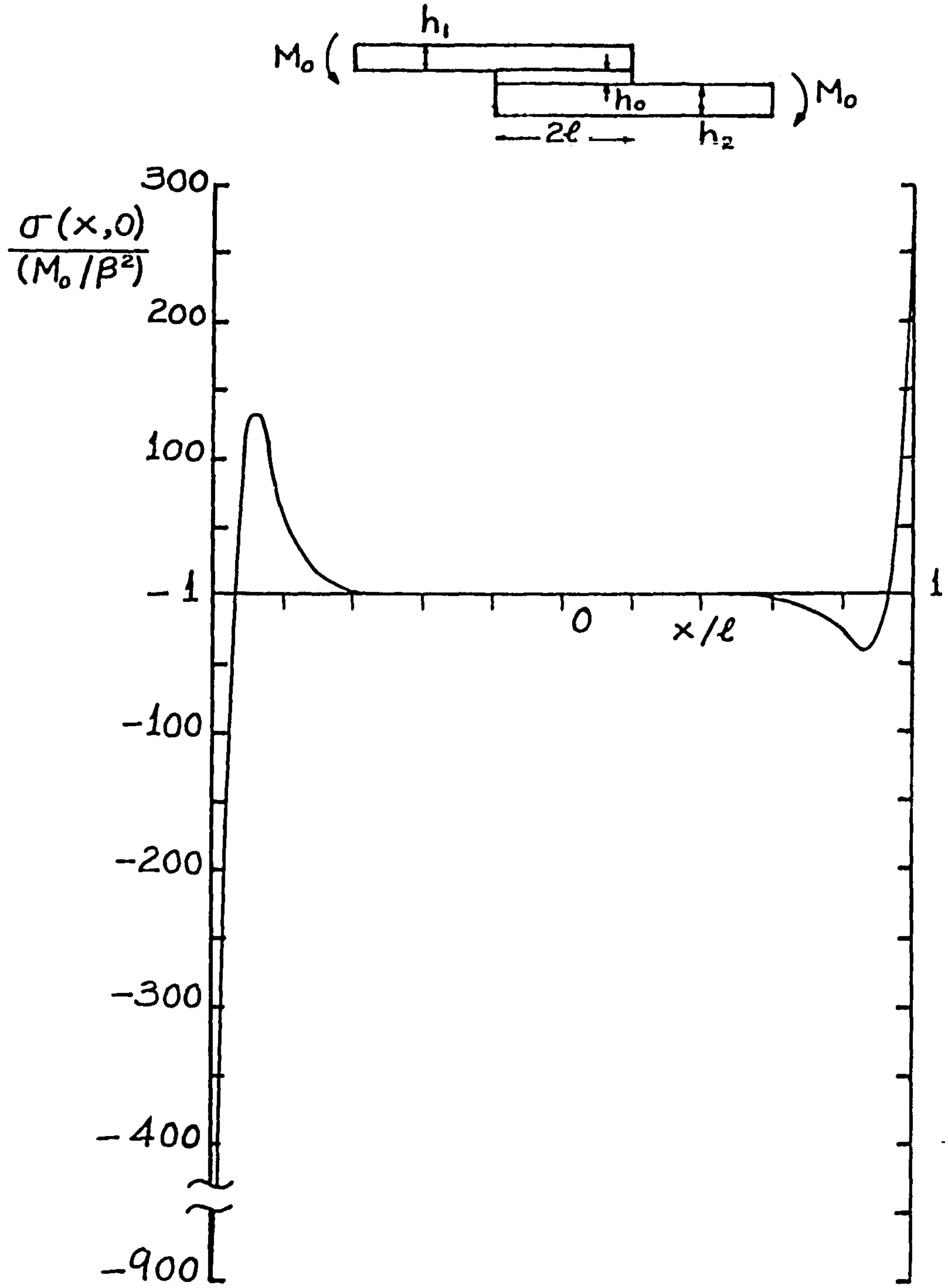


Figure 7. Distribution of normal stress in a single lap joint subjected to bending where h_1=1.27mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, T=21°C, and β=2.54x10^{-2}m.

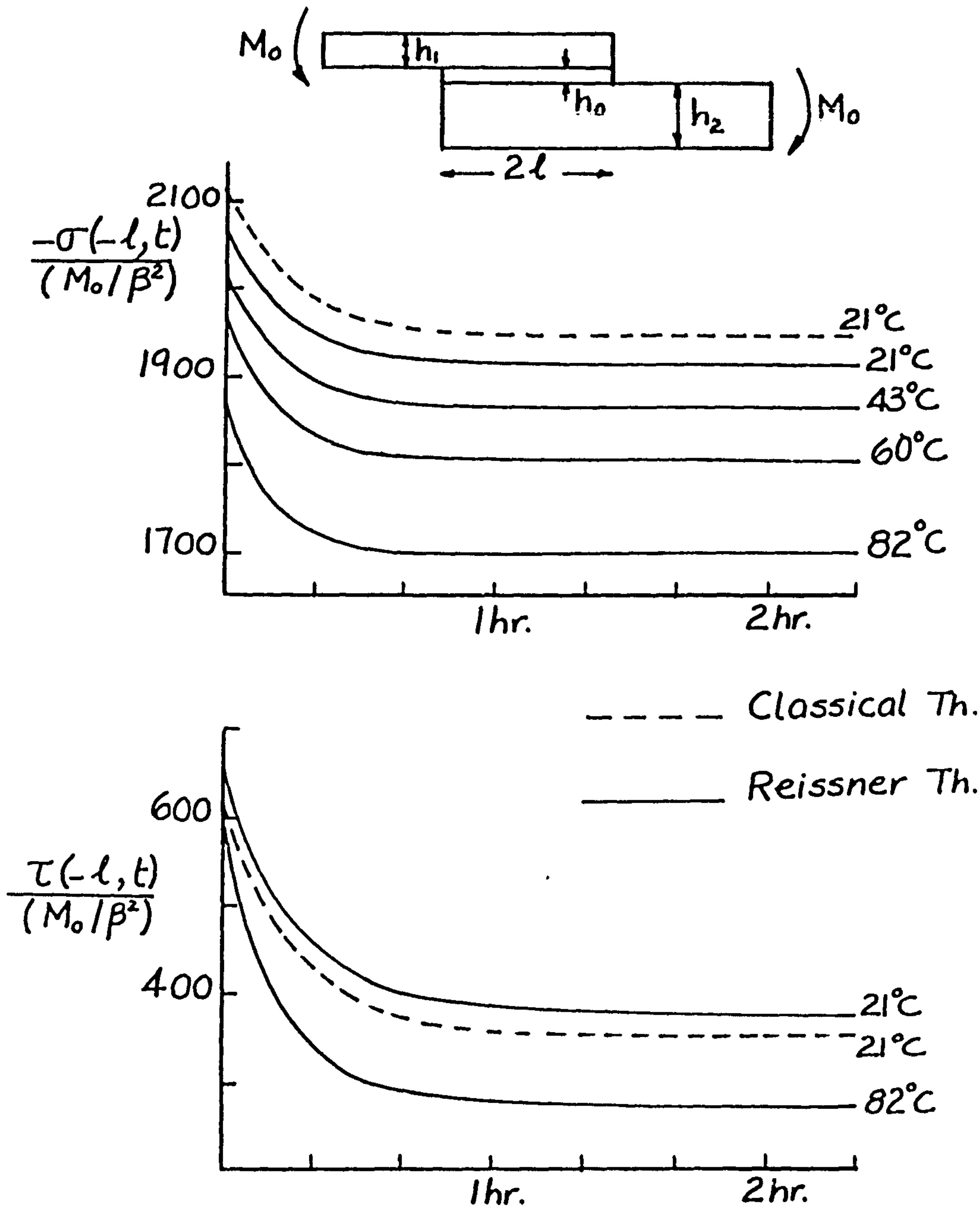


Figure 8. Relaxation of the peak adhesive stresses in a single lap joint subjected to bending at various operating temperatures, where $h_1=.762$mm, $h_2=2.287$mm, $h_0=.1016$mm, $\ell=12.7$mm and $\beta=2.54\times10^{-2}$m.

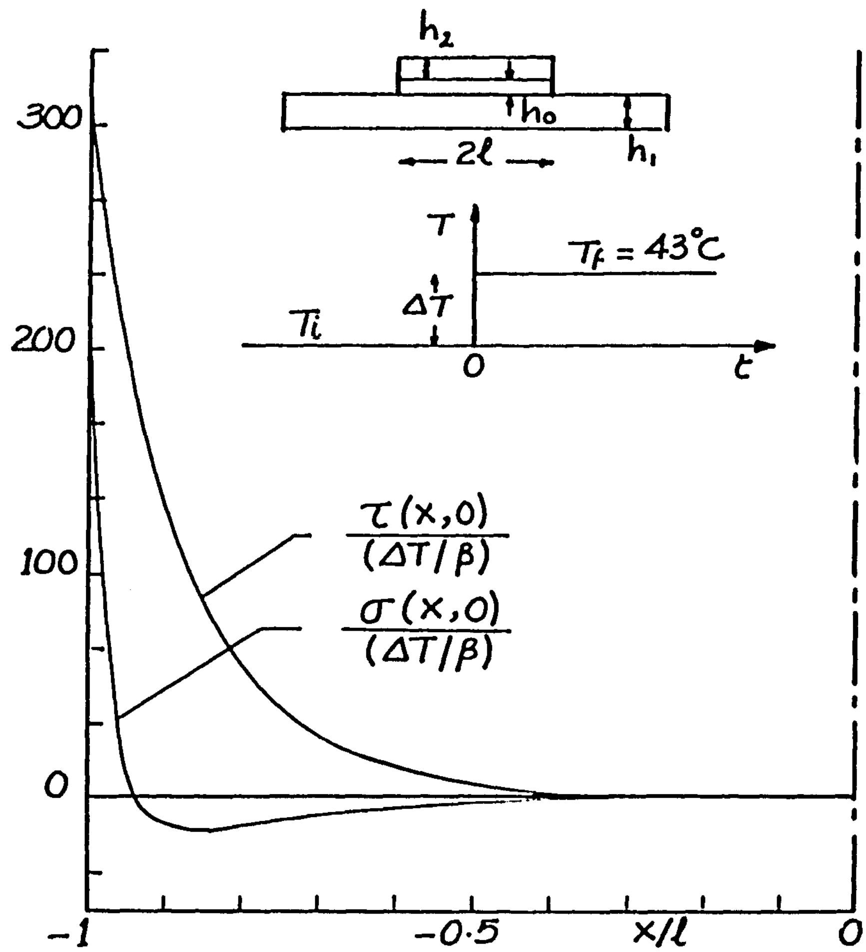


Figure 9. Distribution of the adhesive stresses resulting from a temperature increase in a cover plate where h_1= .762mm, h_2=2.286mm, h_0=.1016mm, ℓ=12.7mm, and β= $(2.54\times10^{-2})^2(5/9°C)/(4.448N)$.

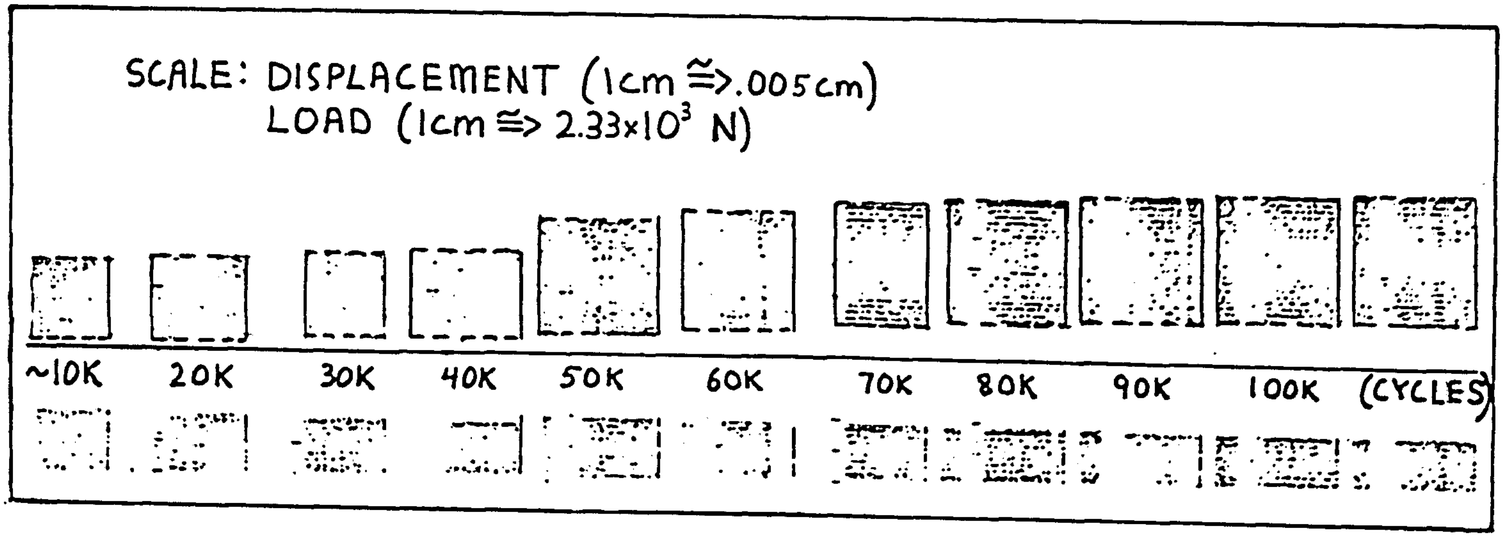
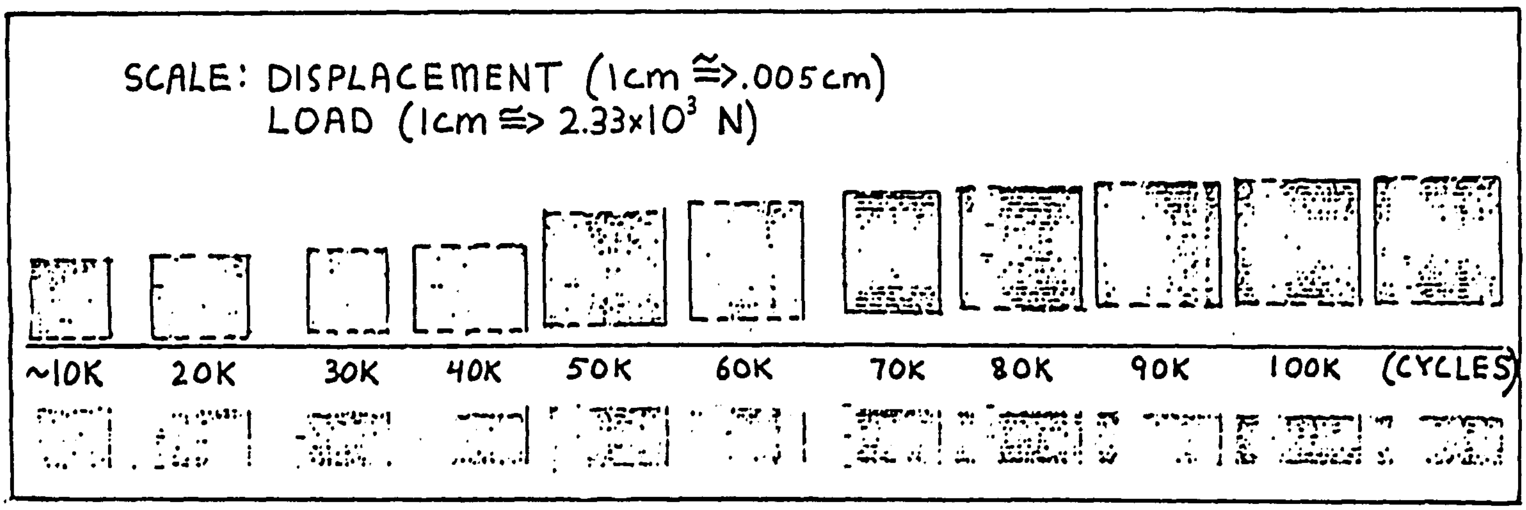


Figure 10. Results from a Nasa test showing increasing displacement amplitude of a cycling viscous material. Recordings of displacement (upper portion) and load (lower portion) were made every 10,000 cycles. Cycling frequency was 10 hertz.



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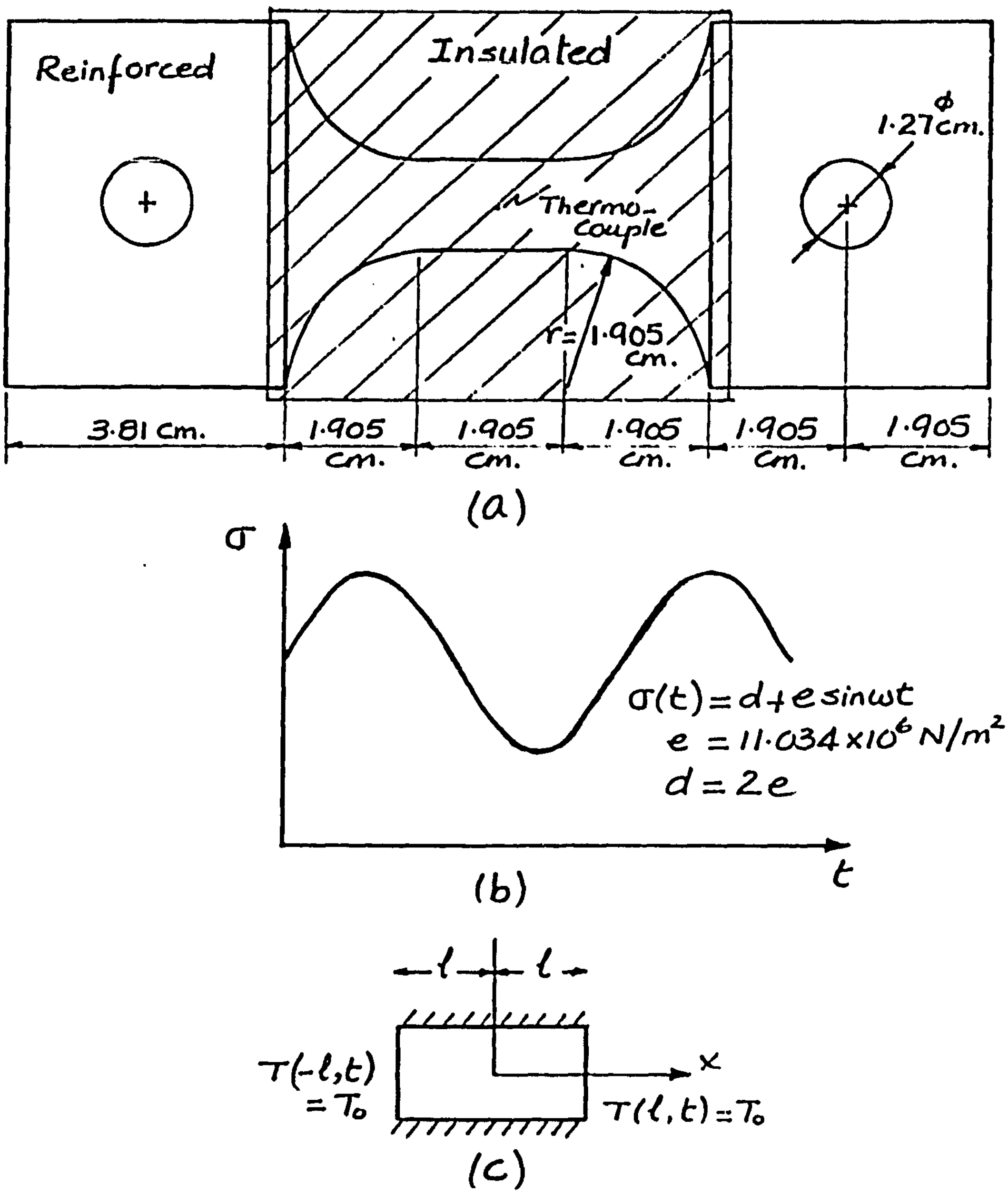


Figure 11. The specimen used in the experiments (a) and the loading for both the theory and the experiment (b). Figure (c) shows the geometry of the model.

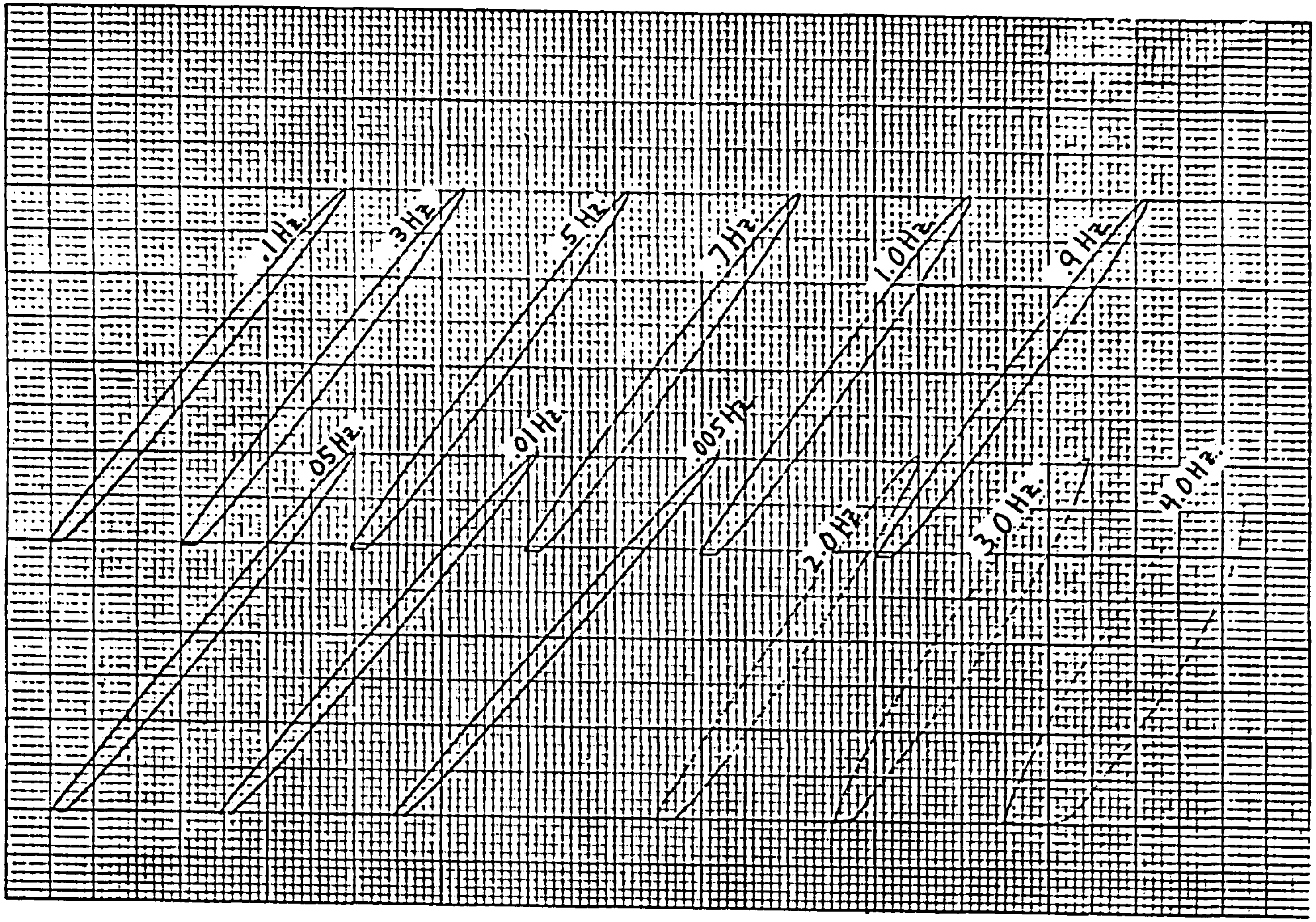


Figure 12. Some hysteresis loops of plexiglas for varying frequencies. The loading is the same as in figure 11b.

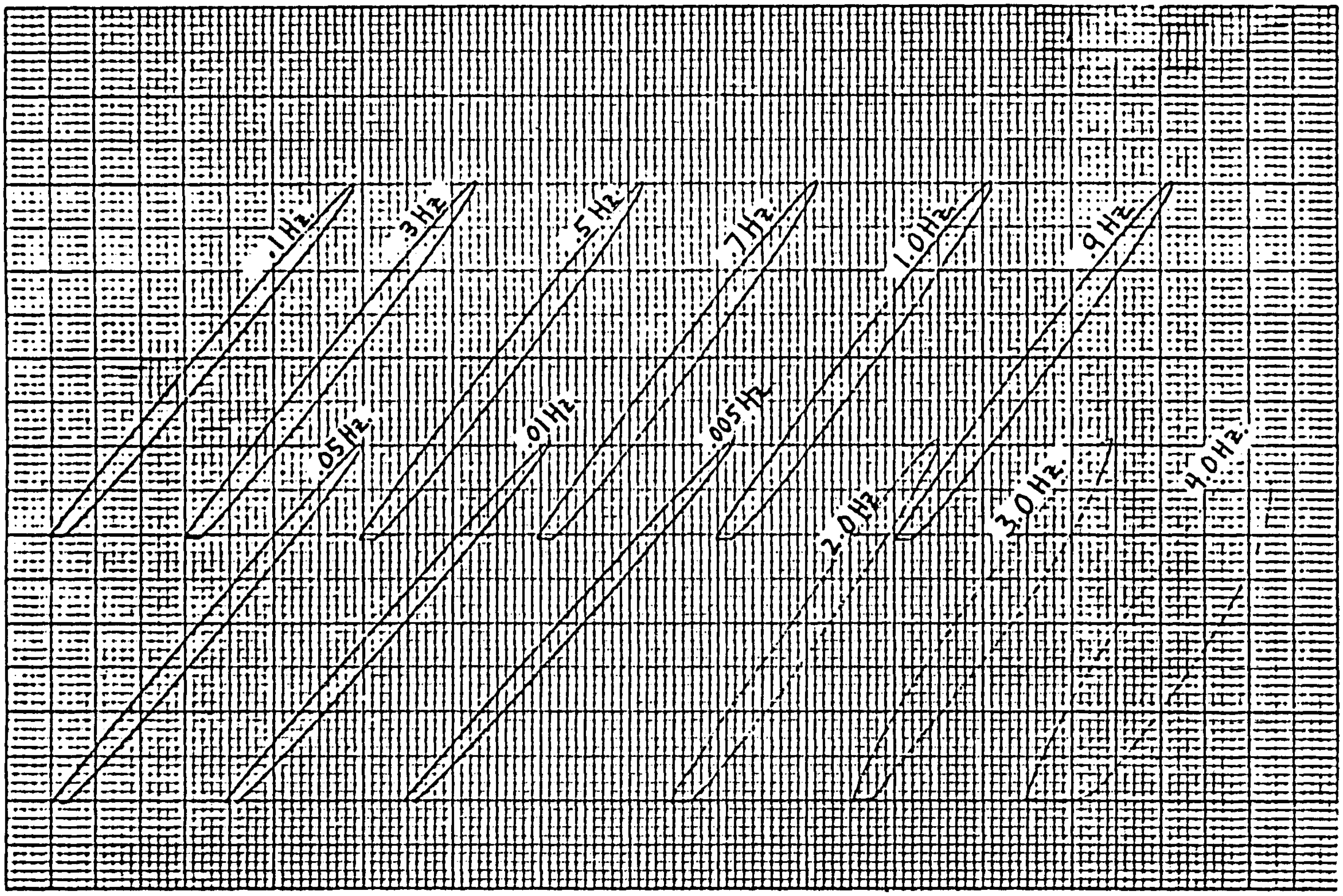
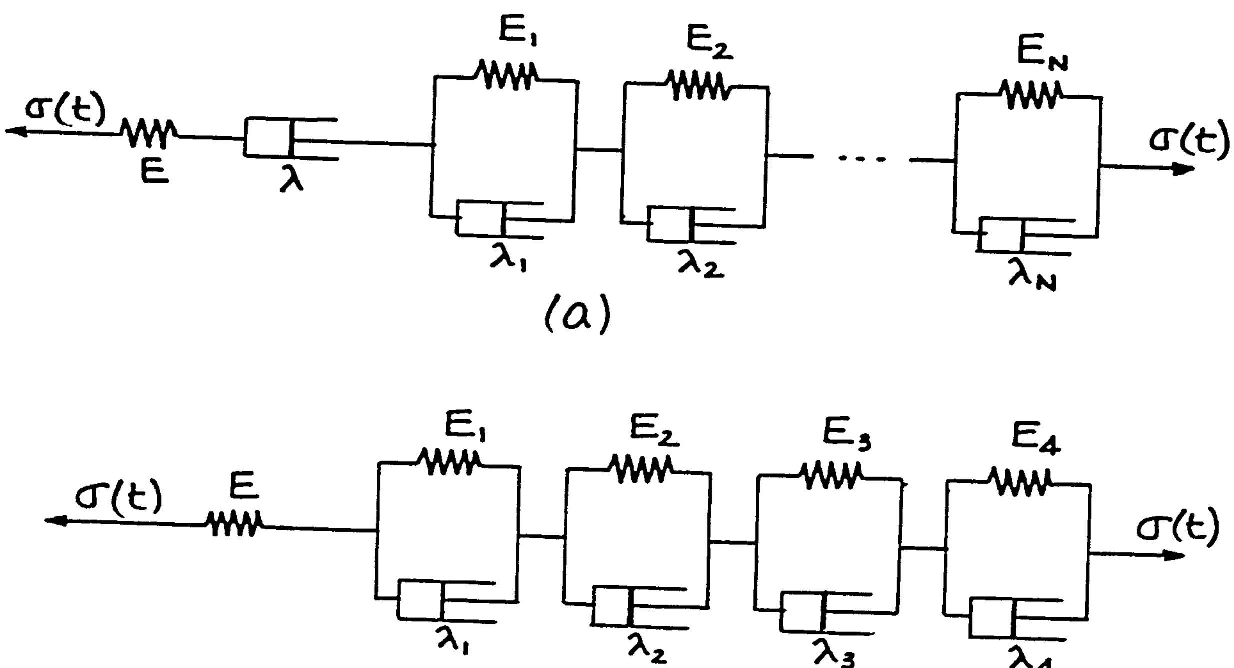


Figure 12. Some hysteresis loops of plexiglas for varying frequencies. The loading is the same as in figure 11b.



$E=2.144\times10^{9} N/m^2$
$E_1=2.439\times10^{9} N/m^2$
$E_2=3.467\times10^{10}$ N/m^2
$E_3=7.724\times10^{10}$ N/m^2
$E_4=8.440\times10^{10}$ N/m^2

$\lambda_1=1.465\times10^{13}$ $N\cdot sec/m^2$
$\lambda_2=1.141\times10^{13}$ $N\cdot sec/m^2$
$\lambda_3=3.457\times10^{11}$ $N\cdot sec/m^2$
$\lambda_4=3.585\times10^{8}$ $N\cdot sec/m^2$

(b)

Figure 13. The generalized Kelvin-Voigt model used to model a viscoelastic material (a). In figure (b) the actual model and constants used to fit the creep curve for plexiglas (figure 14).

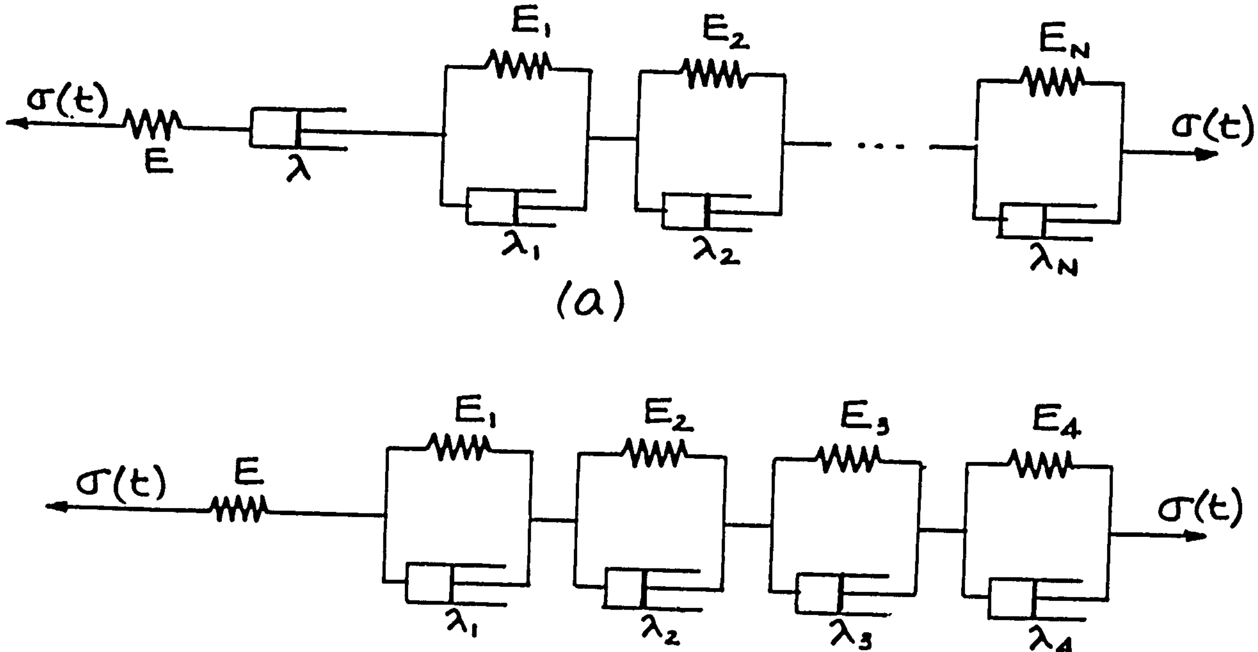


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Figure 13. The generalized Kelvin-Voigt model used to model a viscoelastic material (a). In figure (b) the actual model and constants used to fit the creep curve for plexiglas (figure 14).

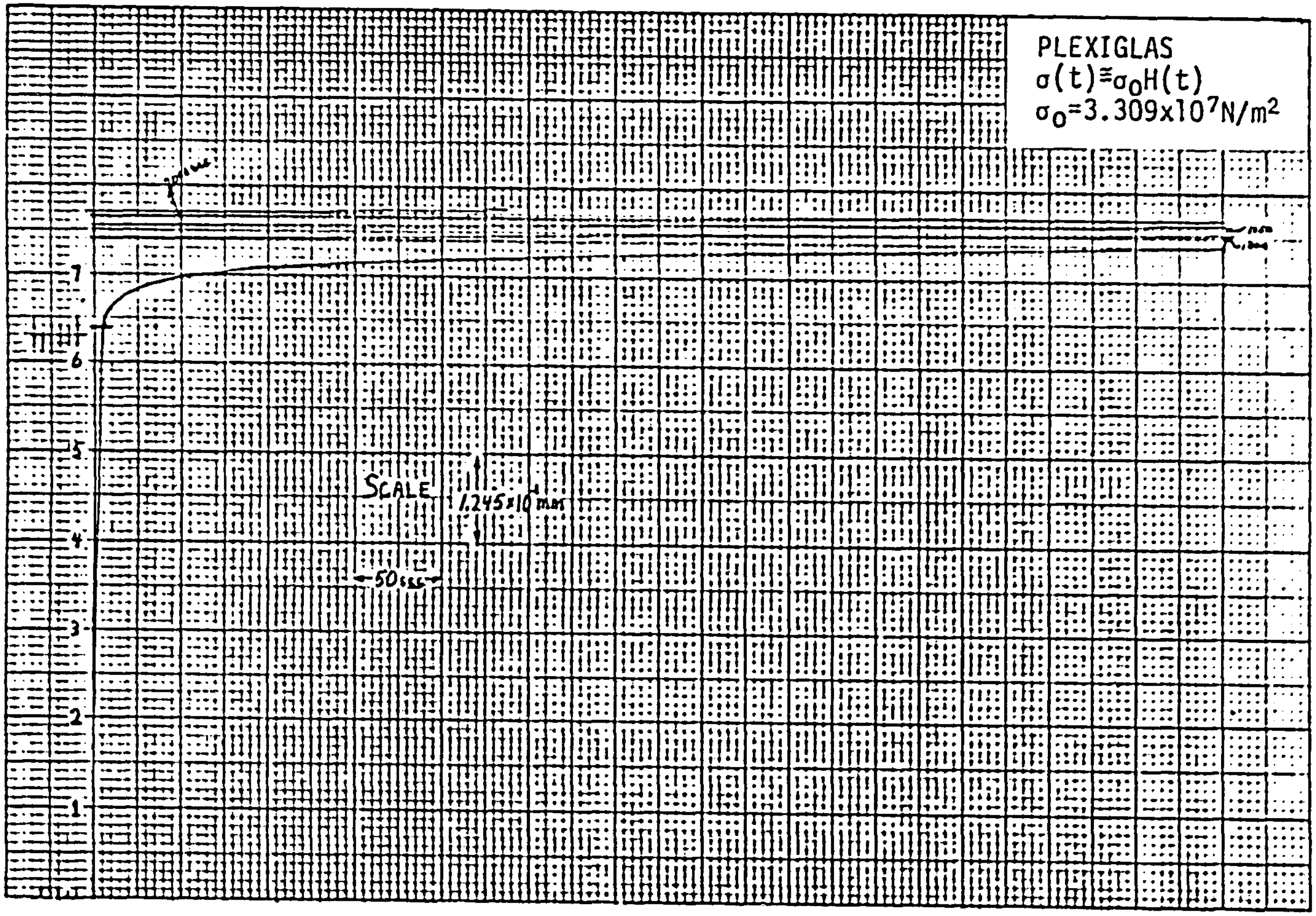

Figure 14. A creep curve for plexiglas.

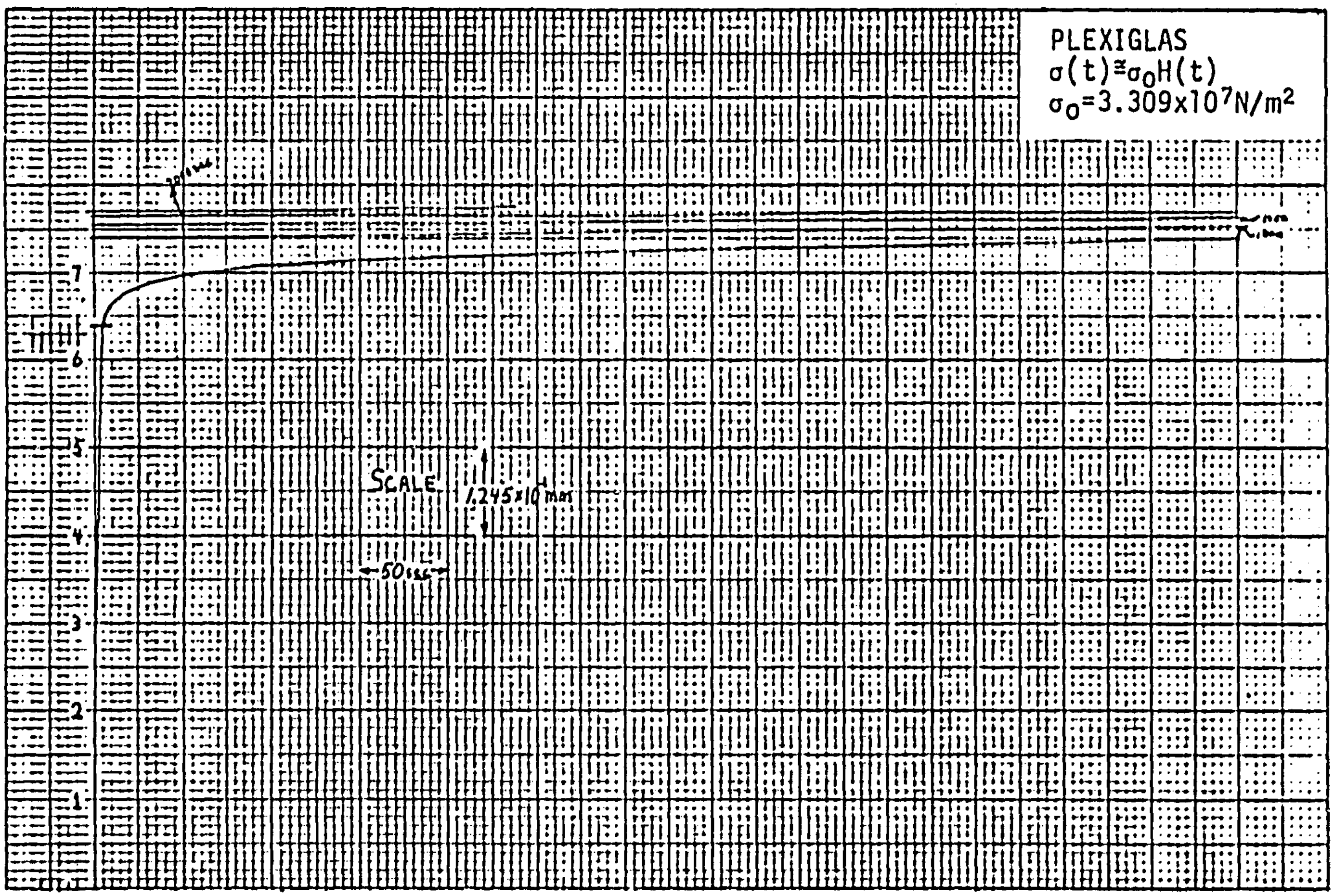

Figure 14. A creep curve for plexiglas.

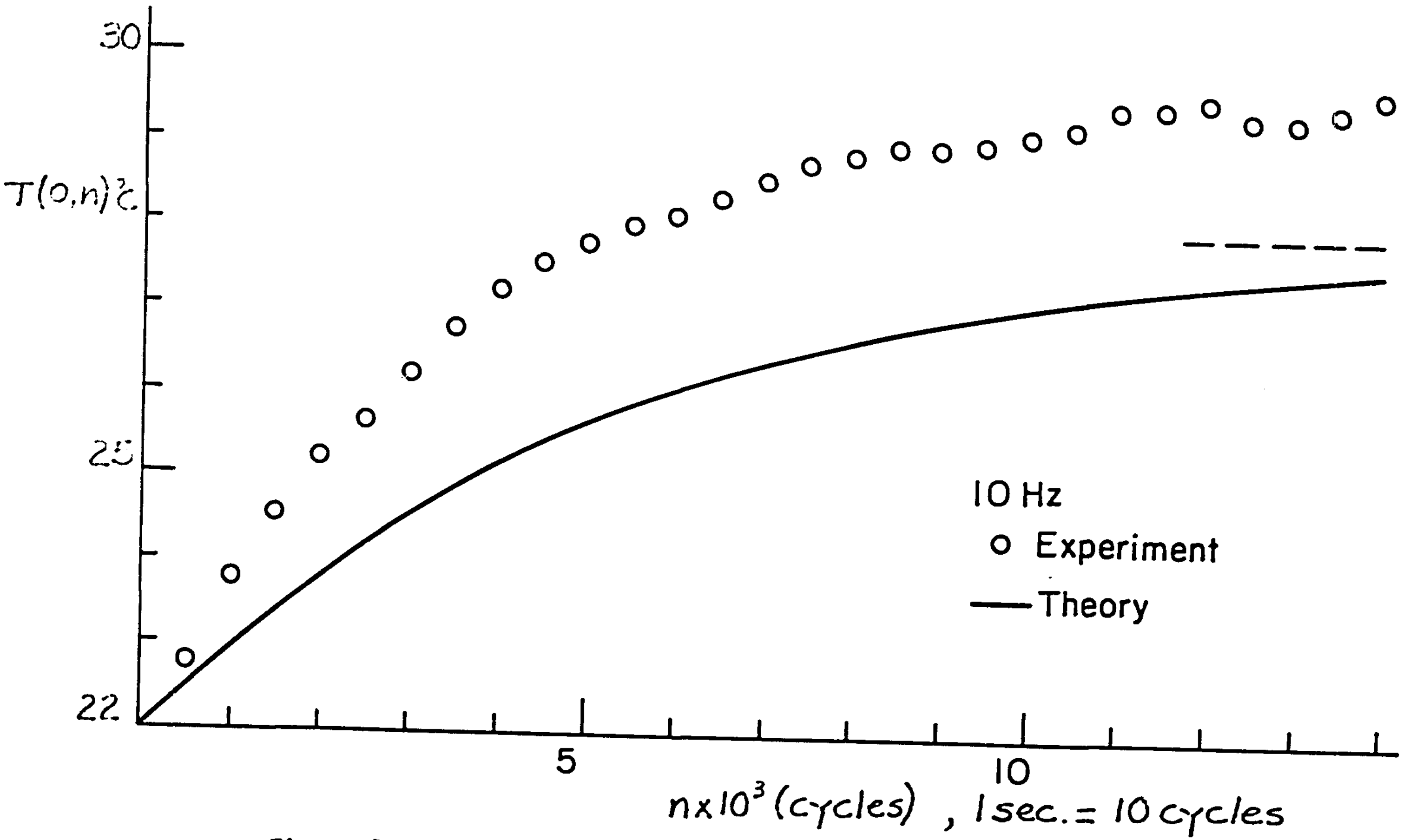

Figure 15. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 10 hz. The material is plexiglas.

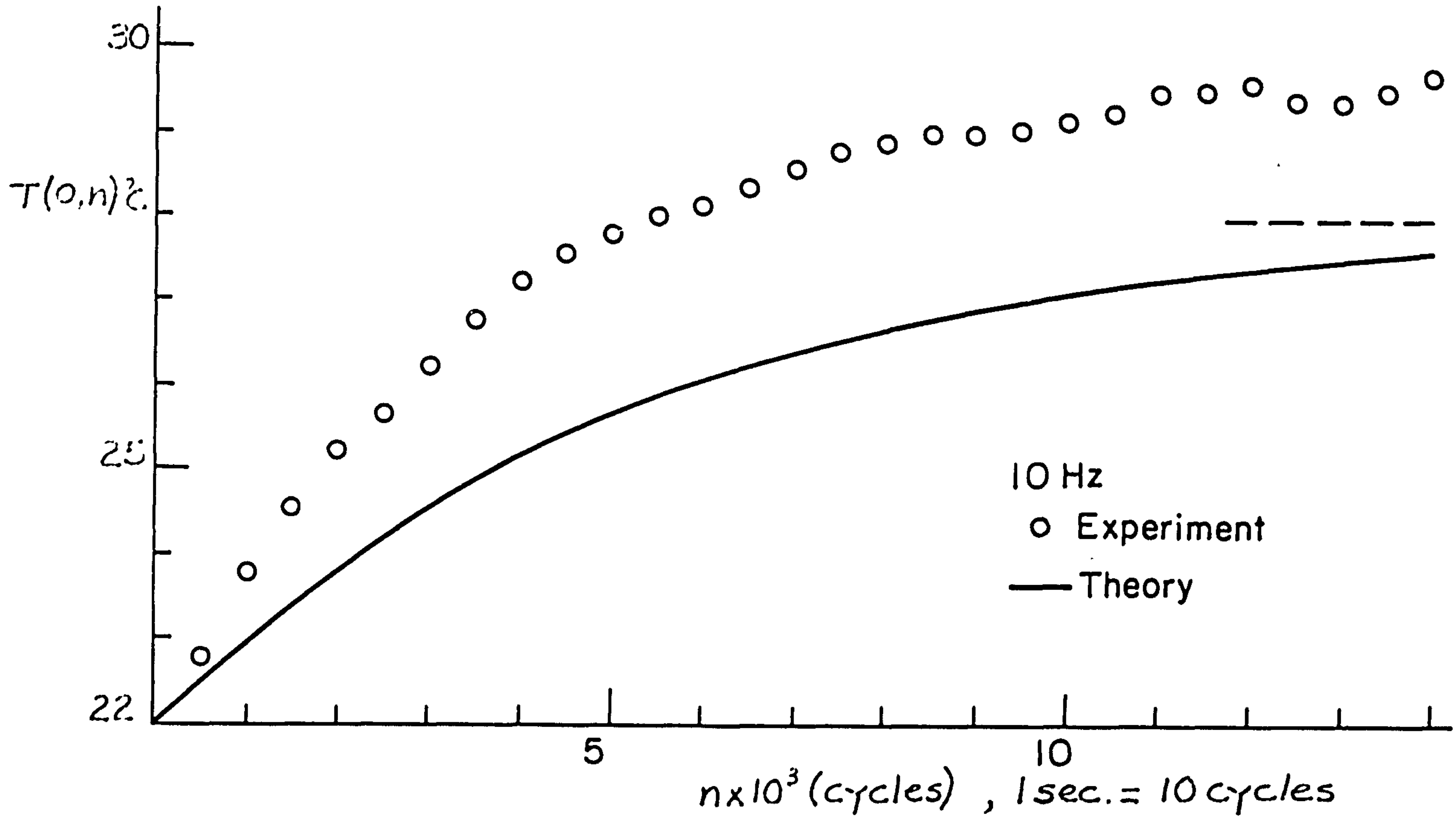

Figure 15. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 10 hz. The material is plexiglas.

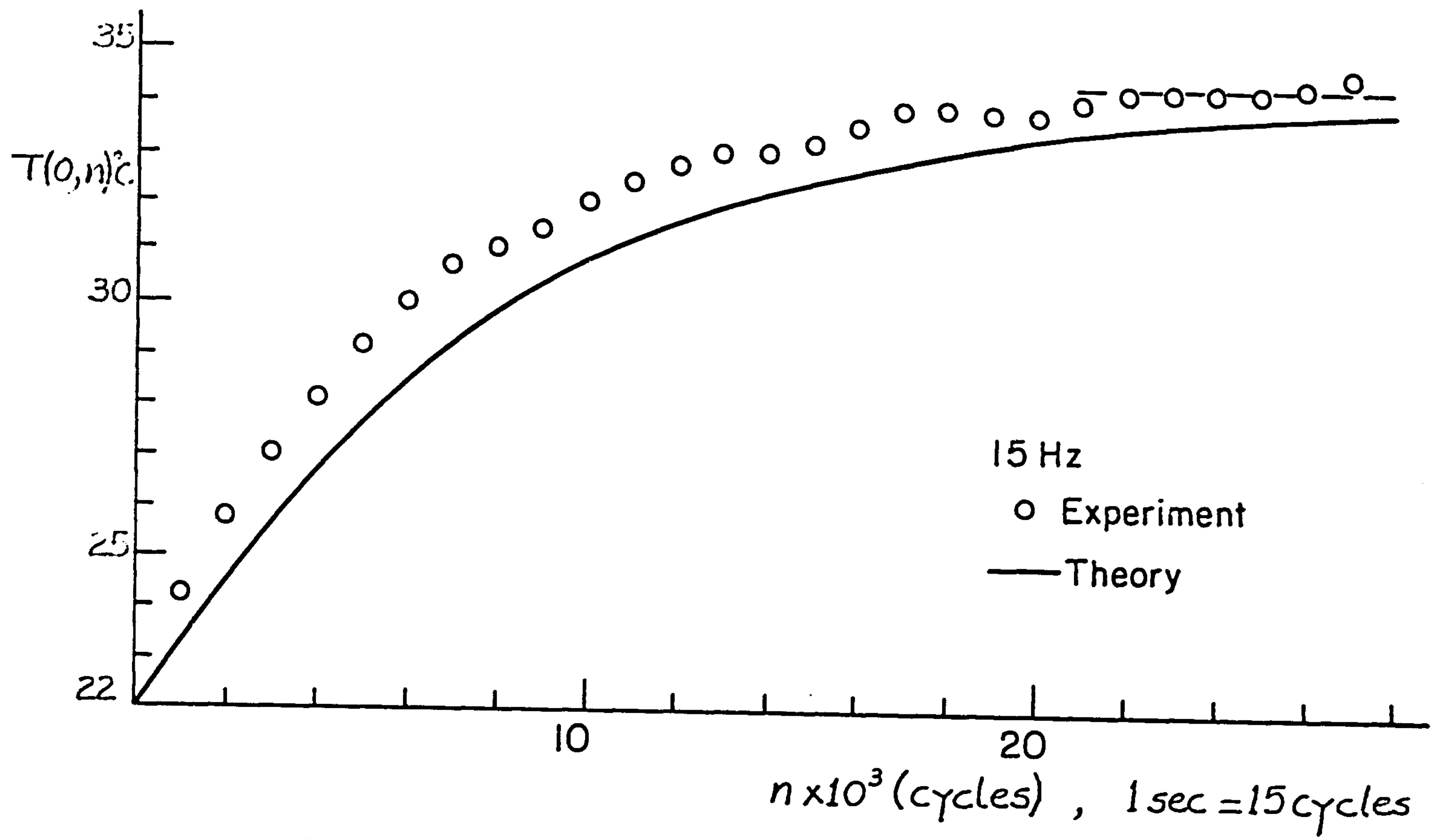

Figure 16. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 15 hz. The material is plexiglas.

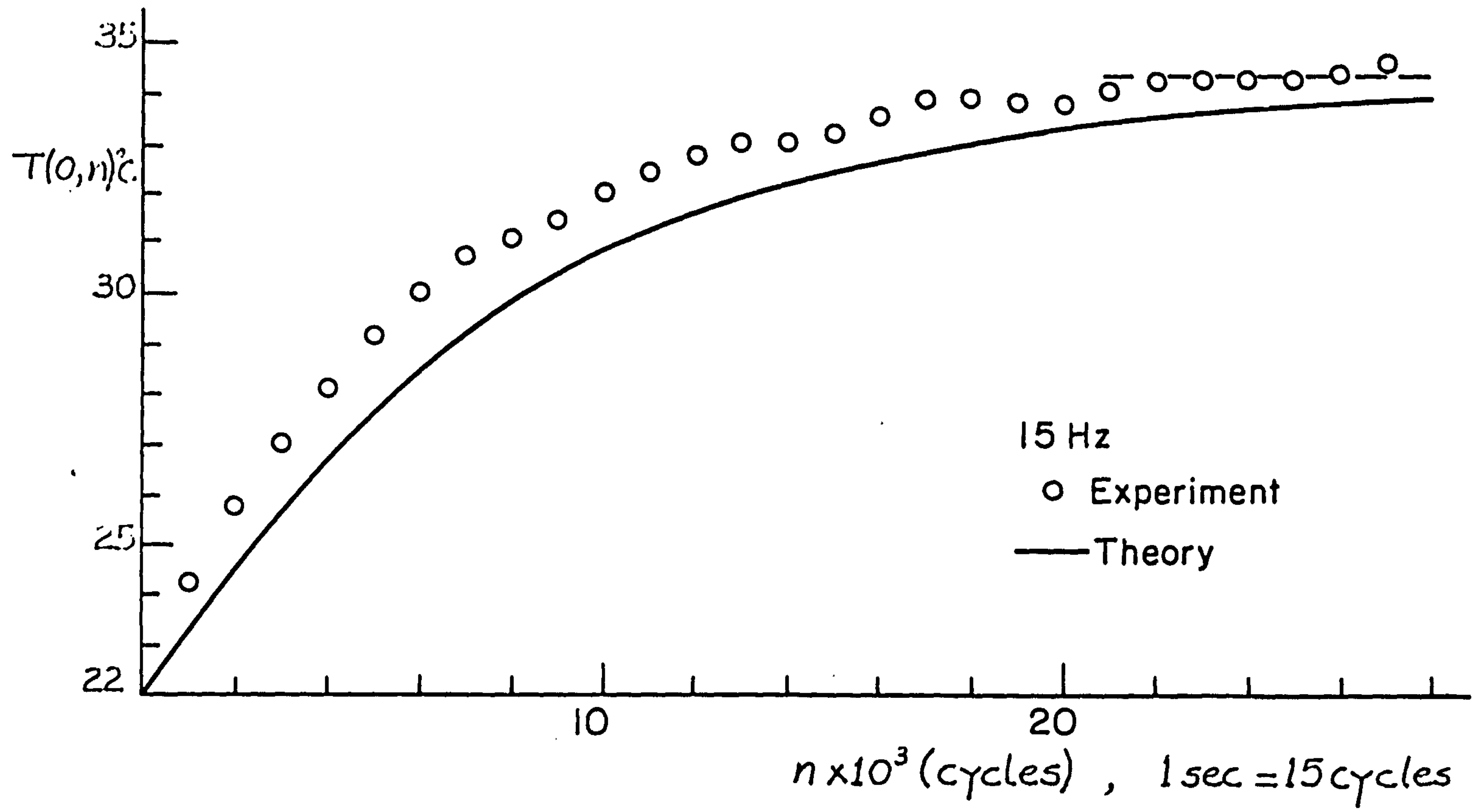

Figure 16. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 15 hz. The material is plexiglas.

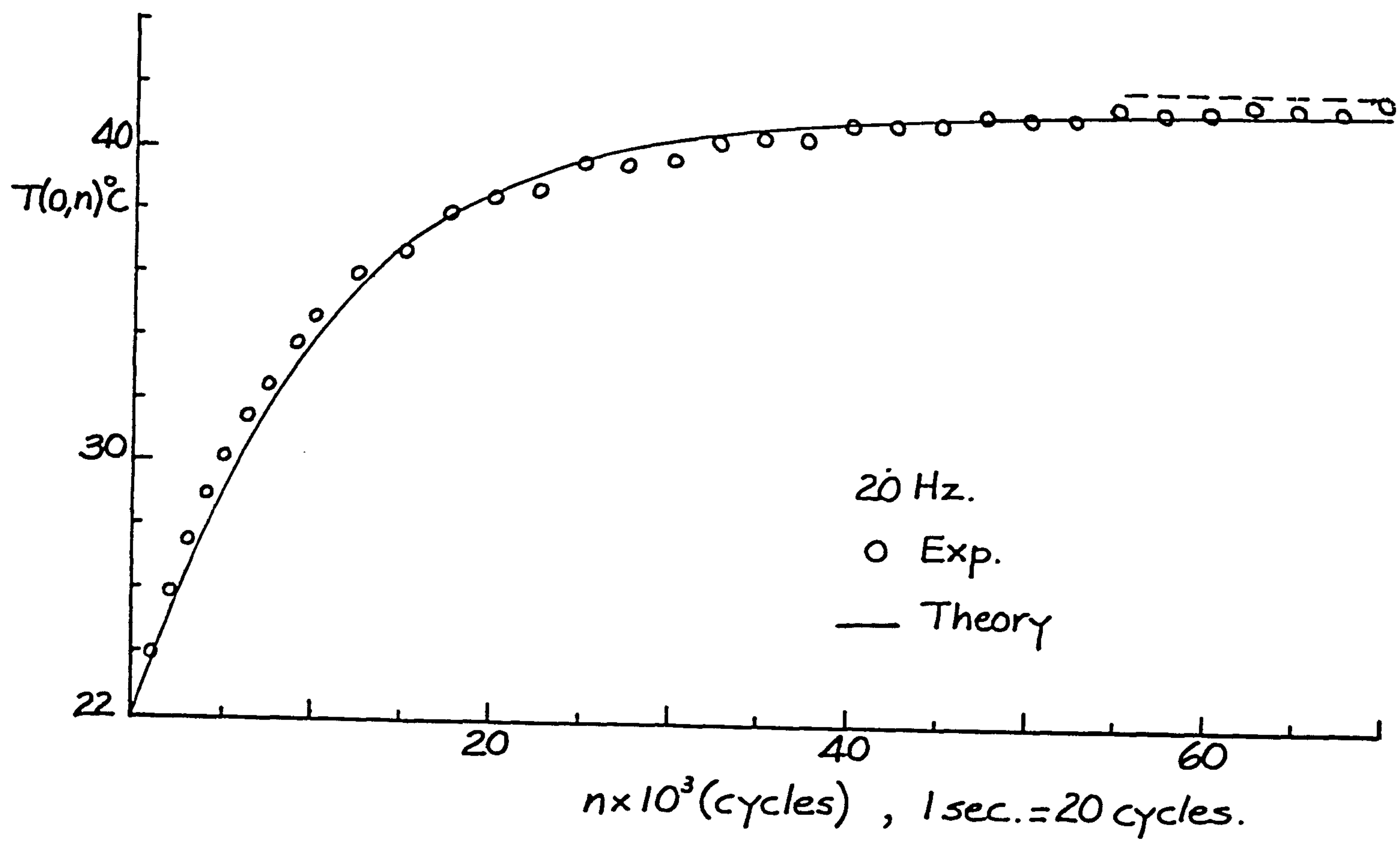

Figure 17. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 20 hz. The material is plexiglas.

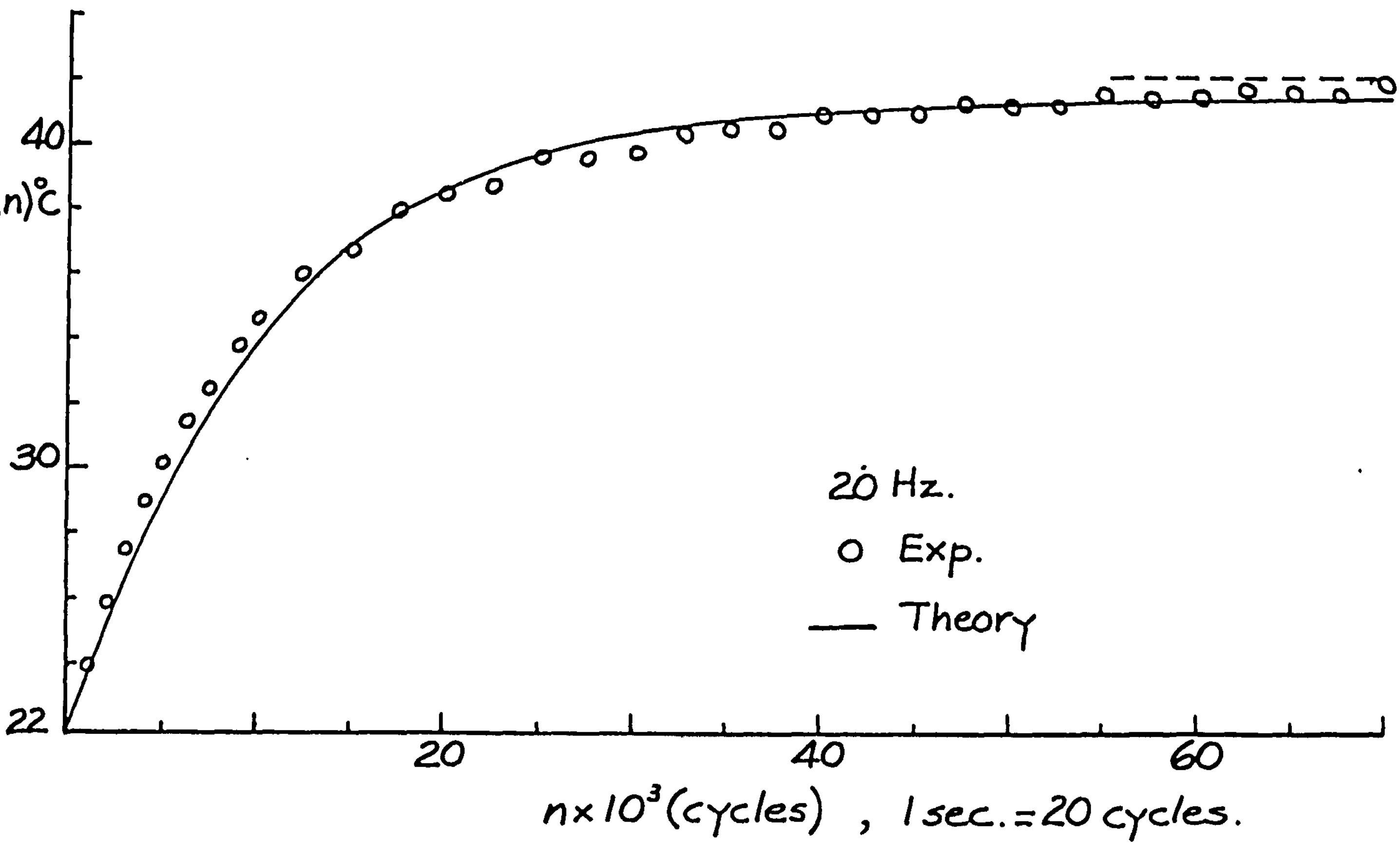

Figure 17. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 20 hz. The material is plexiglas.

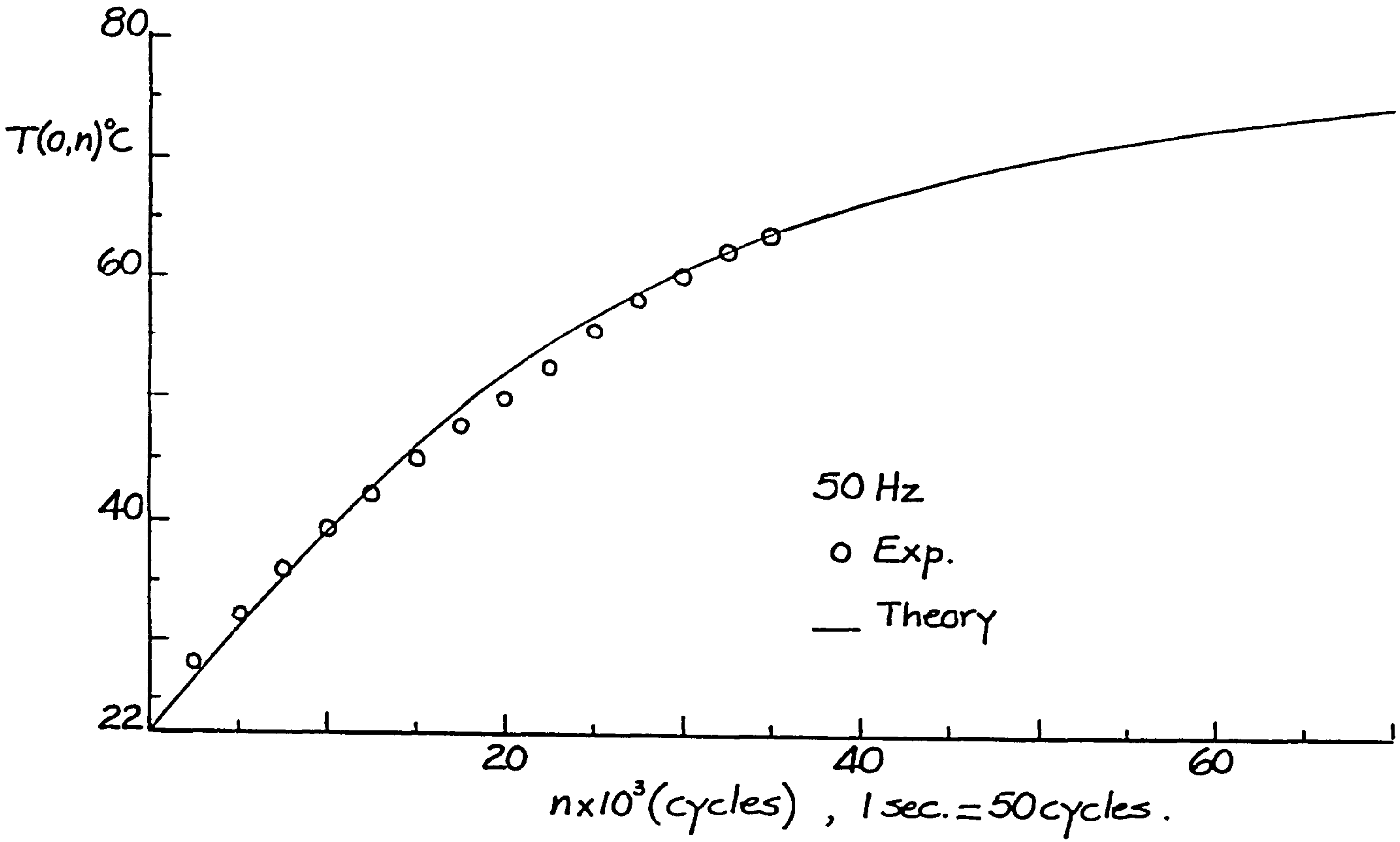

Figure 18. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 50 hz. The material is plexiglas.

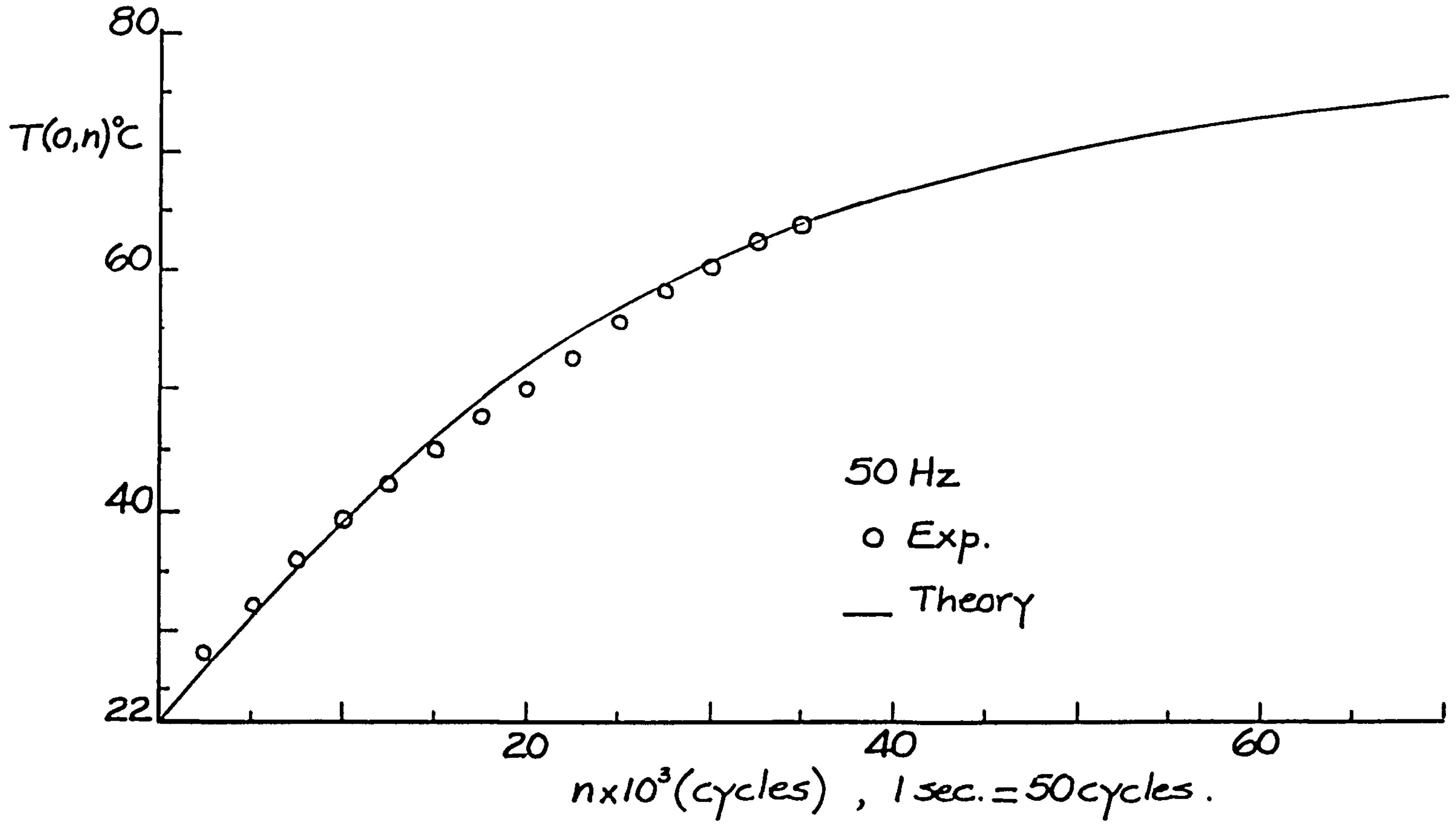

Figure 18. Theoretical and experimental curves for temperature as a function of the number of cycles. The cycling frequency is 50 hz. The material is plexiglas.

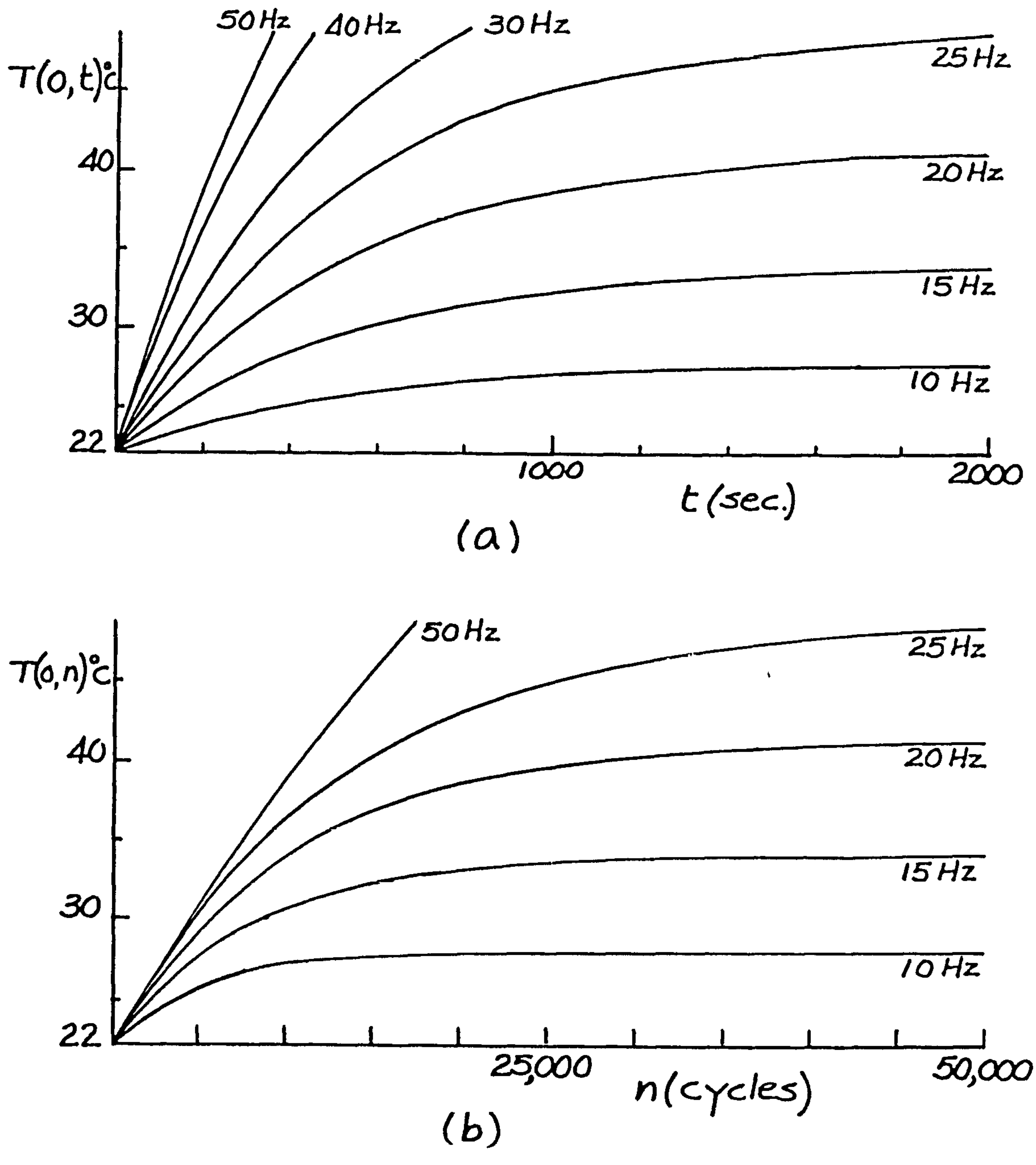

Figure 19. Theoretical curves of temperature versus time (a) and versus number of cycles (b), for various cycling frequencies.

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Appendix A

Least squares fit to creep data.

The creep curve has the form

$$J(t) = \frac{1}{E} + \frac{t}{\lambda} + \sum_{i=1}^{N} \frac{1}{E_i} \left(1 - e^{-\frac{E_i}{\lambda_i} t}\right). \tag{A1}$$

The elastic response of the material is included with the $\frac{1}{E}$ term in (A1). This value is simply J(0) and can be calculated directly from the graph (figure 14). The value of λ is simply the inverse slope of the curve for large time. Plexiglas was assumed to be a solid, therefore λ was taken to be infinite. The remaining constants to be determined in a least squares sense are E_i, λ_i.

Denote the sum of the squares as

$$R(E_i,\lambda_i) = \sum_{m=1}^{L} \left[\sum_{i=1}^{N} \frac{1}{E_i} \left(1 - e^{-\frac{E_i}{\lambda_i} t_m}\right) - D(t_m)\right]^2, \tag{A2}$$

where L is the number of data points used and D(tm) are the values obtained from the curve in figure 14 less the elastic response. We seek the values of E_i, λ_i for a given N that minimizes R. To do this, we try to solve the following:

$$\frac{\partial R}{\partial E_k} = 0 \qquad k = 1,\ldots,N \tag{A3}$$

$$\frac{\partial R}{\partial \lambda_k} = 0 \qquad k = 1,\ldots,N. \tag{A4}$$

These relations give the following equations

$$\sum_{m=1}^{L}\left[\sum_{i=1}^{N}\frac{1}{E_i}\left(1-e^{-\frac{E_i}{\lambda_i}t_m}\right)-D(t_m)\right]\left(\frac{t_m}{\lambda_k}e^{-\frac{E_k}{\lambda_k}t_m}-\frac{1}{E_k}\left(1-e^{-\frac{E_k}{\lambda_k}t_m}\right)\right)=0 \tag{A5}$$

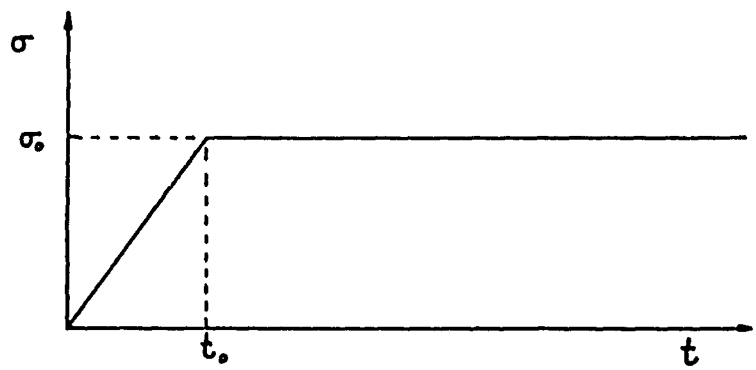
$$\sum_{m=1}^{L}\left[\sum_{i=1}^{N}\frac{1}{E_i}\left(1-e^{-\frac{E_i}{\lambda_i}t_m}\right)-D(t_m)\right]t_m e^{-\frac{E_k}{\lambda_k}t_m}=0 \qquad k=1,\ldots,N. \tag{A6}$$

Because it is difficult to solve this system of 2N non-linear equations, the following successive approximation scheme is used.

First an initial guess was made for the constants. Then E_1 was determined according to equation (A3). This value replaced the guessed, initial value of E_1. Then E_2 was determined using (A3). Again this value replaced the initial value of E_2. This procedure was continued up till N after which λ_1 was determined. It was found that the sum of the squares decreased after each application of either equations (A3) or (A4). The iterations were stopped after the change from one iteration to another was minimal. For the curve of figure 14, I used N=4 (see figure 13b).

Appendix B

In order to obtain more accurate information about the small time behavior of a creep curve, it is suggested that a ramp load be applied initially instead of an attempt to experimentally duplicate the unit step function. The loading is shown below,



and given by the expression

$$\sigma(t) = H(t)\,\frac{\sigma_0}{t_0}\,t + \sigma_0 H(t-t_0)\left(1 - \frac{t}{t_0}\right). \tag{B-1}$$

With this as an input we use the Hereditary integral to obtain the strain.

$$\varepsilon(t) = \sigma(t)J(0) + \int_0^t \sigma(t')\,\frac{dJ(t-t')}{d(t-t')}\,dt'. \tag{B-2}$$

After substituting and integrating we obtain for $t < t_0$

$$\varepsilon(t) = \frac{1}{E}\frac{\sigma_o}{t_o} t\, H(t) + \frac{\sigma_o}{2t_o\lambda} t^2 H(t)$$

$$+ H(t)\frac{\sigma_o}{t_o}\sum_{i=1}^{N}\frac{1}{E_i}\left[t - \frac{\lambda_i}{E_i} + \frac{\lambda_i}{E_i} e^{-\frac{E_i}{\lambda_i}t}\right], \qquad \text{(B-3)}$$

for $t > t_o$

$$\varepsilon(t) = \frac{\sigma_o}{E} + \frac{\sigma_o}{t_o}\sum_{i=1}^{N}\frac{1}{E_i}\frac{\lambda_i}{E_i} e^{-\frac{E_i}{\lambda_i}t}$$

$$+ \frac{\sigma_o}{\lambda}(t-t_o) + \sum_{i=1}^{N}\frac{1}{E_i}\left(1-e^{-\frac{E_i}{\lambda_i}(t-t_o)}\right)$$

$$+ \frac{\sigma_o}{2\lambda}t_o + \frac{\sigma_o}{t_o}\sum_{i=1}^{N}\left\{\frac{1}{E_i}t_o e^{-\frac{E_i}{\lambda_i}(t-t_o)} - \frac{1}{E_i}\frac{\lambda_i}{E_i} e^{-\frac{E_i}{\lambda_i}(t-t_o)}\right\}.$$

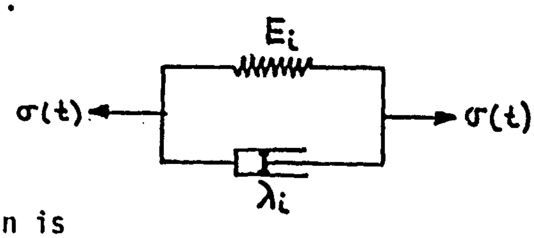
(B-4)

The creep compliance is

$$J(t) = \frac{\varepsilon(t)}{\sigma_o}. \qquad \text{(B-5)}$$

Appendix C

Here the equations of motion are determined using an alternate method. Consider a spring and dashpot in parallel and subjected to a load $\sigma(t)$.



The governing equation is

$$\sigma(t) = E_i \varepsilon_i(t) + \lambda_i \varepsilon_i(t). \tag{C1}$$

Note here that

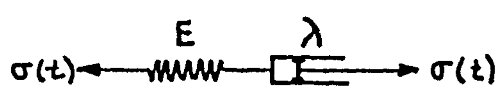
$$\dot{\varepsilon}_{\lambda i}(t) = \dot{\varepsilon}_i(t) \;,\quad \sigma_{\lambda i}(t) = \lambda_i \dot{\varepsilon}_i(t), \tag{C2}$$

for $\sigma(t) = d + e\sin\omega t$ we find

$$\dot{\varepsilon}_i(t) = \left[\frac{e\lambda_i\omega}{\lambda_i{}^2\omega^2 + E_i{}^2} - \frac{d}{E_i} \right] e^{-\frac{E_i}{\lambda_i}t} + \frac{d}{E_i} + \frac{E_i e}{\lambda_i{}^2\omega^2 + E_i{}^2} \sin \omega t$$

$$- \frac{e\lambda_i\omega}{\lambda_i{}^2\omega^2 + E_i{}^2} \cos\omega t. \tag{C3}$$

Now consider a spring and dashpot in series.



The governing equation is

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \frac{\sigma(t)}{\lambda} \quad . \tag{C4}$$

Note here that

$$\sigma_\lambda = \sigma(t) \quad , \qquad \dot{\varepsilon}_\lambda = \sigma(t)/\lambda \ . \tag{C5}$$

Solving this equation for

$$\sigma(t) = d + e \sin \omega t, \tag{C6}$$

we obtain

$$\varepsilon(t) = \frac{d}{E} + \frac{e}{E} \sin \omega t + \frac{d}{\lambda} t - \frac{e}{\lambda\omega} \cos \omega t + \frac{e}{\lambda\omega} \ . \tag{C7}$$

Since the spring and dashpot system shown in figure 13a is linear, simply add (C-7) and each of the N components of (C-3) to get the identical result obtained from equation (123).

$$\text{i.e.} \quad \varepsilon_T(t) = \varepsilon(t) + \sum_{i=1}^{N} \varepsilon_i(t) \quad . \tag{C8}$$

Appendix D

Solution of the partial differential equation

$$\frac{\partial T}{\partial t} = a\,\frac{\partial^2 T}{\partial x^2} + Q_0 e^{bt} \tag{D1}$$

subject to the conditions (see figure 11c)

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \tag{D2}$$

$$T(\pm\ell, t) = T_0 \tag{D3}$$

$$T(x,0) = T_0\,. \tag{D4}$$

Define

$$\bar{T}(x,s) = \int_0^\infty T(x,t)e^{-st}dt \tag{D5}$$

$$T(x,t) = \int_{c-i\infty}^{c+i\infty} \bar{T}(x,s)e^{st}ds\,. \tag{D6}$$

After taking the Laplace Transform of (D-1), (D-2) and (D-3), we obtain

$$s\,\bar{T}(x,s) - T(x,0) = a\,\frac{\partial^2 \bar{T}}{\partial x^2} + Q_0\,\frac{1}{s-b} \tag{D7}$$

$$\left.\frac{\partial \bar{T}}{\partial x}\right|_{x=0} = 0 \tag{D8}$$

$$\bar{T}(\pm\ell, t) = \frac{T_0}{s}\,. \tag{D9}$$

Rearranging D7 we find

$$a \frac{\partial^2 \bar{T}}{\partial x^2} - s\ \bar{T}(x,s) = -(Q_o \frac{1}{s-b} - T_o) \tag{D10}$$

which has the solution

$$\bar{T}(x,s) = A(s)\sinh\sqrt{\frac{s}{a}}\,x + B(s)\cosh\sqrt{\frac{s}{a}}\,x + \frac{Q_o}{s(s-b)} + \frac{T_o}{S}\ . \tag{D11}$$

After applying the transformed boundary conditions we obtain

$$\bar{T}(x,s) = \frac{Q_o}{s(s-b)} \frac{\cosh\sqrt{\frac{s}{a}}\,x}{\cosh\sqrt{\frac{s}{a}}\,\frac{\ell}{2}} + \frac{Q_o}{s(s-b)} + \frac{T_o}{s}\ , \tag{D12}$$

from the inversion integral we can determine T(x,t);

$$T(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \{ \frac{Q_o}{s(s-b)} \frac{\cosh\sqrt{\frac{s}{a}}\,x}{\cosh\sqrt{\frac{s}{a}}\,\frac{\ell}{2}} + \frac{Q_o}{s(s-b)} + \frac{T_o}{s} \} e^{st} ds. \tag{D13}$$

Now to calculate the residues. There are contributions from 0, b, and from the zeros of $\cosh\sqrt{\frac{s}{a}}\,\frac{\ell}{2}$, which occur at

$$s_j = -\frac{(2_j+1)^2 a\pi^2}{\ell^2}\ . \tag{D14}$$

The residue at zero is

$$\frac{-2\ Q_0}{b} + T_0\ , \tag{D15}$$

from b

$$\left[\ \frac{Q_0}{b} \frac{\cosh \sqrt{\frac{b}{a}}\, x}{\cosh \sqrt{\frac{b}{a}}\, \frac{\ell}{2}} + \frac{Q_0}{b}\ \right] e^{bt}\ , \tag{D16}$$

and from s_j

$$-\sum_{j=1}^{\infty} \frac{4Q_0 \ell (-1)^j}{\pi(2j+1)} \cos \frac{(2j+1)\pi x}{\ell}\ e^{-s_j t}\ . \tag{D17}$$

After collecting all contributions the solution is found to be

$$T(x,t) = T_0 + \frac{Q_0}{2a}\left[\left(\frac{\ell}{2}\right)^2 - x^2\right] - \sum_{j=0}^{\infty} \frac{4Q_0}{a} (-1)^j\ \frac{\ell^2}{\pi^3}\, x$$

$$+ \frac{1}{(2j+1)^3} \cos \frac{(2j+1)\pi x}{\ell}\ e^{-s_j t}\ , \tag{D18}$$

for the special case of b=0, we get

$$T(x,t) = T_0 + \frac{Q_0}{2a}\left[\left(\frac{\ell}{2}\right)^2 - x^2\right]$$

$$-\sum_{j=0}^{\infty} \frac{Q_0 4}{a} (-1)^j\ \frac{\ell^2}{\pi^3} \frac{1}{(2j+1)^3} \cos\left[(2j+1)\pi\ \frac{x}{\ell}\right] e^{-s_j t}\ . \tag{D19}$$

Since the partial differential equation is linear we can add solutions for the case where there are several values of b occurring as the nonhomogeneous part of (D1).

Vita

The author was born in Princeton, New Jersey on January 27, 1958 to Dr. and Mrs. John Joseph. He received his Bachelor of Arts degree in June 1979 from Franklin and Marshall College in Lancaster, Pennsylvania with a major in Physics. In September of 1979 he began graduate studies at Lehigh University in the Mechanical Engineering Department.